Solution key to HW Chap.1

1. $y(x) = -3x^4$.
2. $y(x) = \sin[e^x - 1 + \frac{\pi}{6}]$.
3. $y(x) = \tan(\tan^{-1}x + \tan^{-1}3 - \tan^{-1}2)$.
4. $y(x) = 1 - \sqrt{8 + e^x}$.
5. $\frac{1}{y} + \ln(1 - \frac{1}{y}) = \cos x + C.$
6. $y(x) = \frac{-x^2}{x - C}$.
7. $y(x) = -x \ln(\ln x - C)$.
8. $y(x) = x \ln(1 + Cx^2)$.
9. $y(x) = x \sinh(C + \ln x)$.
10. let v = ay+bx+k, dy/dx = dy/dv $\frac{dy}{dx} = F(ay + bx + c) = F(v - k + c) = G(v)$

    $$\frac{dy}{dx} = \frac{dy}{dv}\frac{dv}{dx} = \frac{dy}{dv}(ay' + b) = \frac{dy}{dv}[aF(ay + bx + c) + b] = \frac{dy}{dv}[aG(v) + b] = G(v)\ .$$

    Therefore separable.
11. 1^{st} order.
12. 2^{nd} order.
13. 4th order.
14. 3rd order.
15. $y(x) = C - \cos(x^3)$.
16. $y(x) = 2\exp(-2x) + \exp(x) + C_1 + C_2 x$.
17. $y(x) = -5$.
18. $y(x) = 2x^2 - 3x - 17$.
19. $y(x) = \exp(-3x) + 5$.
20. $y(x) = 1 - \cos(x)$.
21. $given\ \ f(1) = 2\ ,\ f'(x) = x^2 + f^2(x) \Rightarrow f'(1) = 1^2 + 2^2 = 5$

    $$\because\ f''(x) = 2x + 2f(x)f'(x) \Rightarrow f''(1) = 2 + 2\times 2\times 5 = 22.$$
    $$\because\ f'''(x) = 2 + 2f'(x)f'(x) + 2f(x)f''(x) \Rightarrow f'''(1) = 2 + 2\times 5^2 + 2\times 2\times 22 = 140.$$
22. $y' = 2 \Rightarrow y(x) = 2x + C, \because\ y(0) = 0 = C\ \ but\ \ y(1) = 100 = 2 + C \Rightarrow C = 98$

    get 2 values for C, this is not an initial value problem for inconsistent initial conditions.

23. Radioactive decay rate is modeled as $\frac{dx(t)}{dt} = -kx(t)$ with given initial condition x(0) = x_0 and with half-life T that x(T) = x_0/2. The above equation can be solved as $\int_{x_0}^{x} \frac{dx}{x} = \int_0^t -kdt \Rightarrow \ln\left(\frac{x}{x_0}\right) = -kt \Rightarrow x(t) = x_0 \exp(-kt)$. With x(T) = x_0/2, k can be determined as $k = \frac{\ln 2}{T} \Rightarrow x(t) = x_0 \exp\left(-\ln 2 \frac{t}{T}\right) = x_0 2^{-t/T}$.

24. see #23

25. y(x) = $\frac{\exp(-x)}{3}[3\cos(\sqrt{3}x) + \sqrt{3}\sin(\sqrt{3}x)]$.

26. y(x) = $\frac{2\exp(-x/2)}{3}[3\cos(\frac{\sqrt{3}x}{2}) + \sqrt{3}\sin(\frac{\sqrt{3}x}{2})]$.

27. $y(x) = (7+2x)\exp(-2-x)$.

28. $y(x) = \frac{1}{125}(-146 - 180x - 75x^2) + C_1\exp(-3-\sqrt{14}x) + C_2\exp(-3+\sqrt{14}x)$.

29. $y(x) = C_1\exp(3x/2)\cos(\sqrt{11}x/2) + C_2\exp(3x/2)\sin(\sqrt{11}x/2) +$

$$\frac{1}{111}[37\exp(x) + 3\cos(2x) - 18\sin(2x)]\left(\cos^2(\frac{\sqrt{11}x}{2}) + \sin^2(\frac{\sqrt{11}x}{2})\right)$$

30. $y(x) = C_1\cos(x) + C_2\sin(x) + \frac{1}{10}[-60\cos^2(x) + 10x^2\cos^2(x) - 60\sin^2(x) +$

$10x^2\sin^2(x) + \cos^2(x)\sinh(3x) + \sin^2(x)\sinh(3x)]$.

31. $y(x) = C_1\exp[(3-\sqrt{2})x] + C_2\exp[(3+\sqrt{2})x] +$

$$\frac{-656\cos(2x) - 459\cos(3x) - 164\sin(2x) + 51\sin(3x)}{6(35\sqrt{2}-62)(35\sqrt{2}+62)}.$$

32. $y(x) = C_1\cos(2x) + C_2\sin(2x) + \frac{1}{80}[-20x\cos(2x) + 60\cos(x)\cos(2x) -$

$12\cos(2x)\cos(5x) - 10\cos^2(2x)\sin(2x) - 60\sin(x)\sin(2x) +$
$5\cos(2x)\sin(4x) - 12\sin(2x)\sin(5x)]$.