4. The critical magnetic field

The critical current density and the critical magnetic field

The supercurrent density has a limit: J_C

When the superconductor is applied with a magnetic field, a supercurrent is generated so as to maintain the perfect diamagnetism. If the current density needed to screen the field exceeds J_C, the superconductor will lose its superconductivity. This limit of the field strength is called the critical magnetic field H_C.

Note the difference between the flux density, B and field strength, H

Free energy

The magnetisation depends on the applied field and temperature but not the history. $\Rightarrow$ The transition is a reversible process

the Gibbs free energy $g_s(T, H_a)$

is a function of temperature and field.

Why Gibbs free energy?

Because the system is in contact with a thermal reservoir, (*T*=constant) a pressure reservoir (*P*=constant), and a "magnetic-field reservoir"(*H*=constant)

In this case, the system will minimize the Gibbs free energy

Below T_C

The free energy of the superconducting state < The free energy of the normal state $g_s\left(T,0\right) < g_n\left(T,0\right)$

The free energy increases when applied a magnetic field:

$$\Delta g\left(H_a\right) = -\mu_0 \int_0^{H_a} M dH_a$$

Area of the *M*-*H* curve

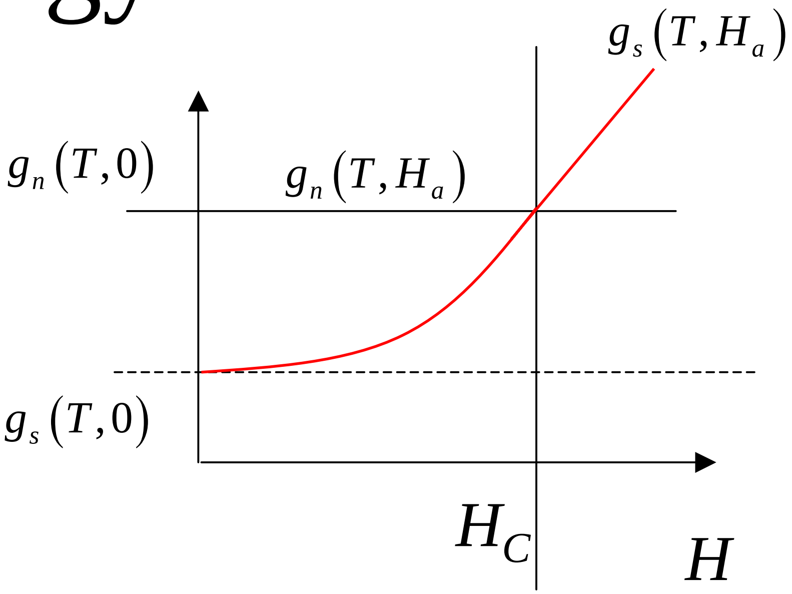
Here we neglect the effect of demagnetizing factor

Free energy

$$g_s(T,H_a) = g_s(T,0) - \mu_0 \int_0^{H_a} M dH_a$$

$$= g_s(T,0) + \mu_0 \int_0^{H_a} H dH_a$$

$$= g_s(T,0) + \mu_0 \frac{H_a^2}{2}$$



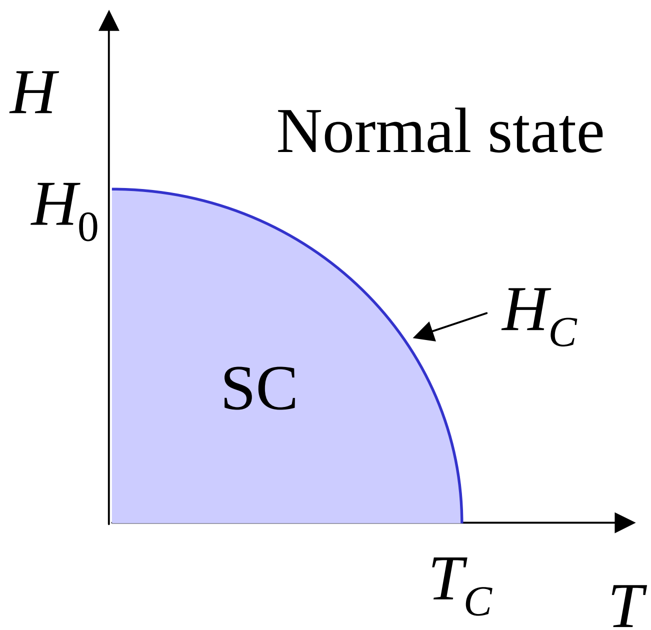
The normal state is non-magnetic and the applied field does not change the free energy much.

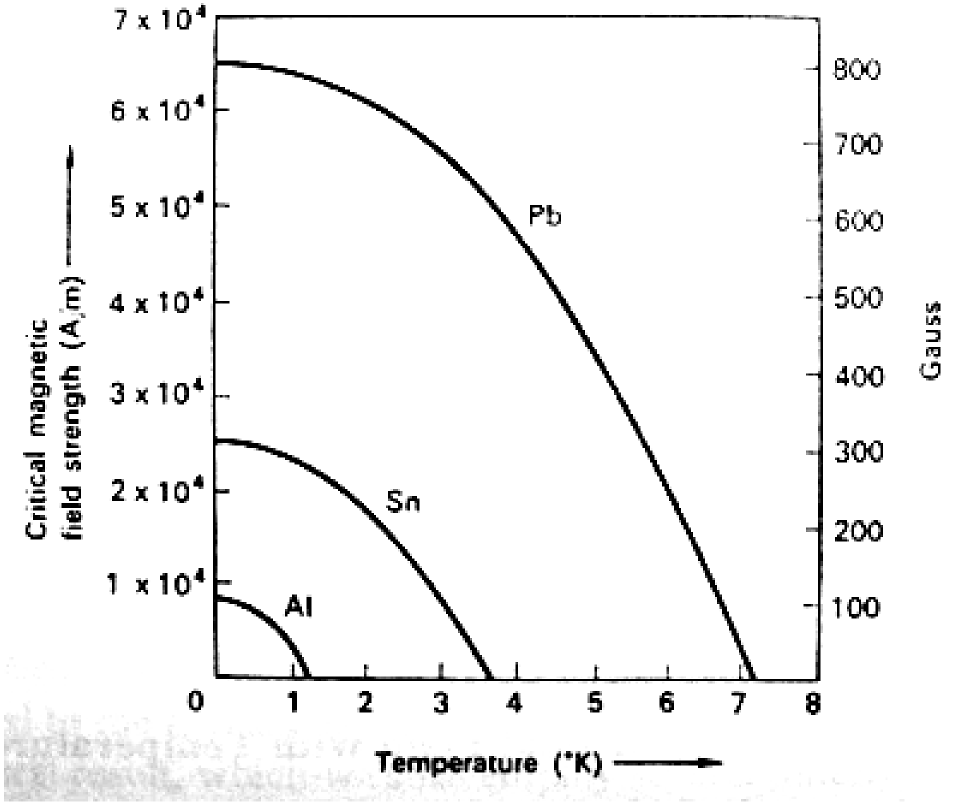
When $g_s(T,H) > g_n(T,H)$ the superconductivity is destryed .

$$\Longrightarrow \quad \mu_0 \frac{H_C^2}{2} = g_n(T,0) - g_s(T,0)$$

The temperature dependence

Phase diagram





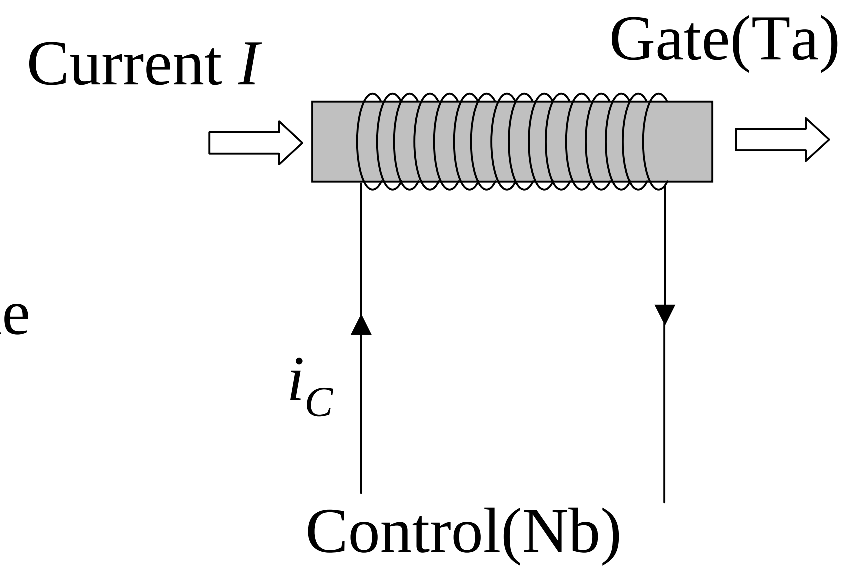
Experimental result:

$$H_C = H_0\left[1-\left(\frac{T}{T_C}\right)^2\right] = H_0\left(1-t^2\right)$$

The cryotron

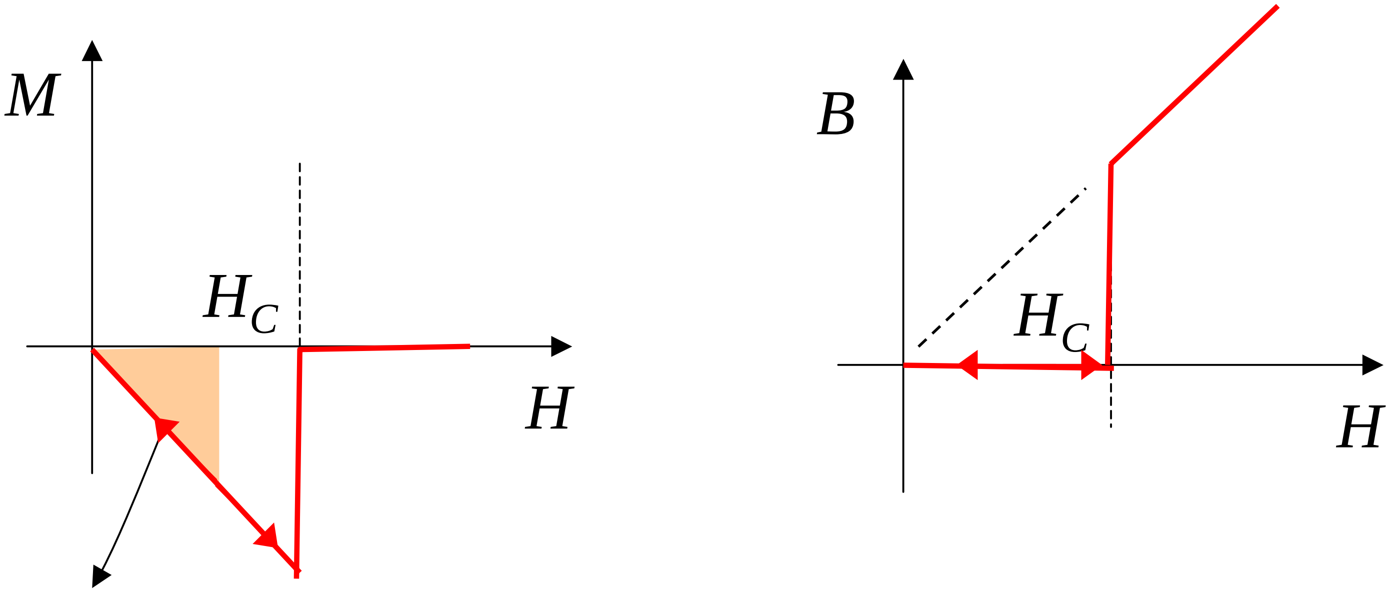
When i_C is applied, the field may destroy the superconductivity of the gate and reduce the current I

An electronic switch



The *M*-*H* curve

Deep inside the superconductor



area: the free energy contributed by *M*

For a perfect specimen, the curves are reversible

The hysteresis: non-ideal specimens

Residual magnetization: trapped flux

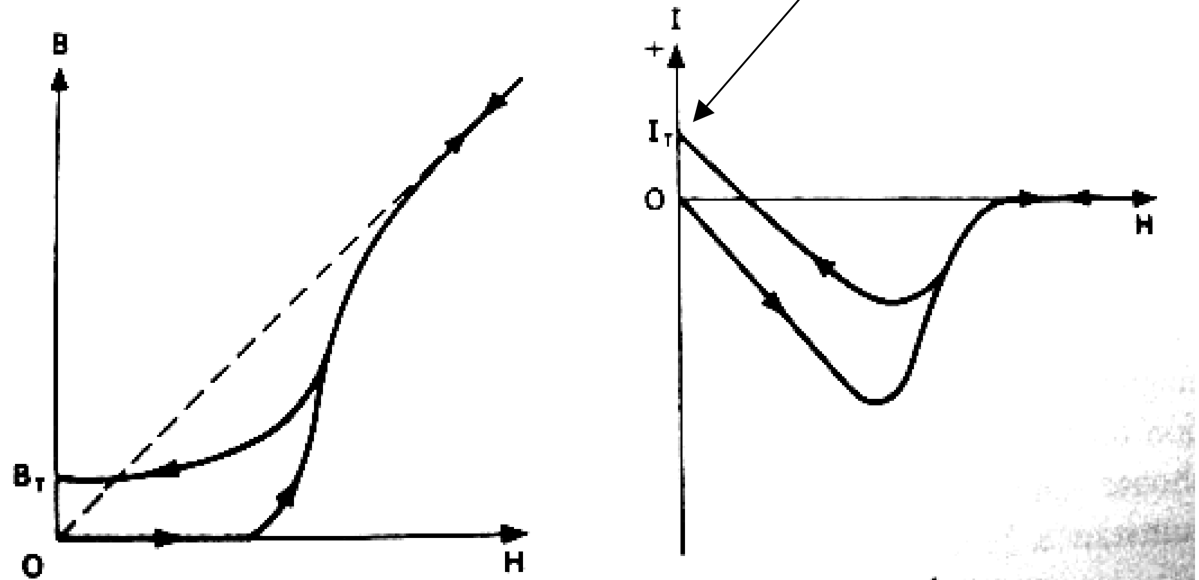


FIG. 4.6. Magnetic behaviour of a non-ideal superconductor.

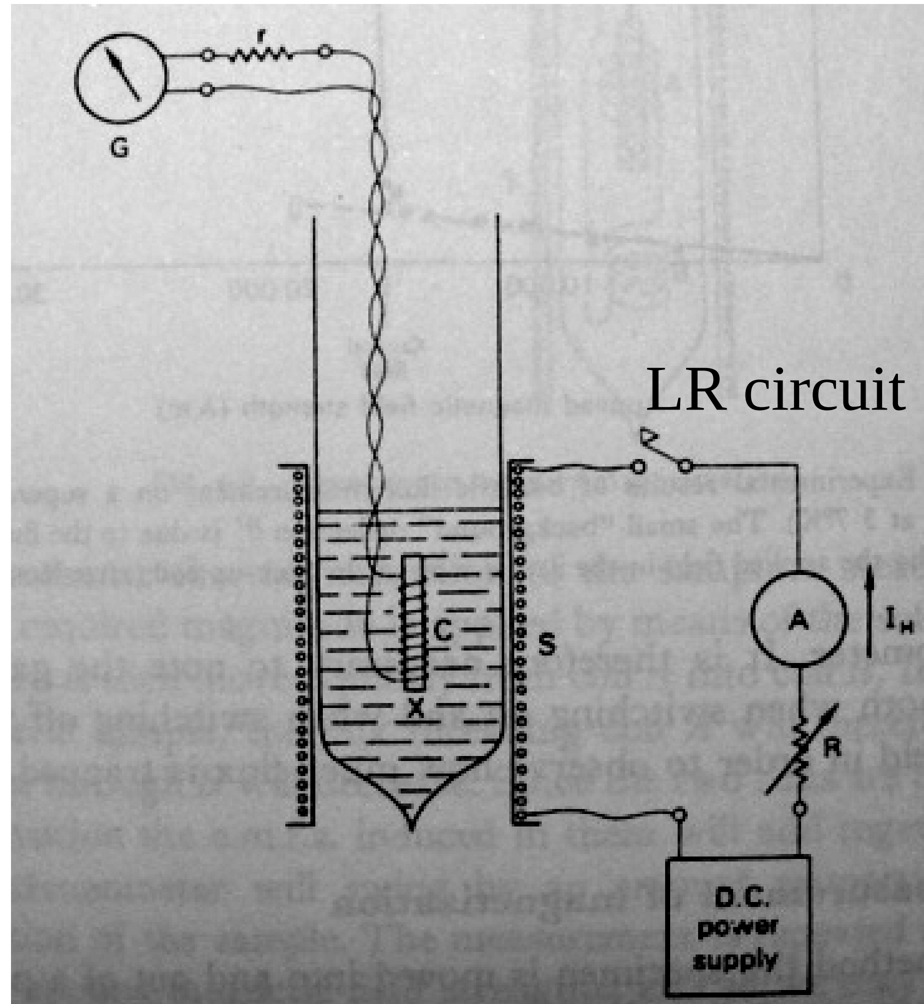
- Ill defined H_C
- Hysteresis
- Trapped flux

Measurement of *B*

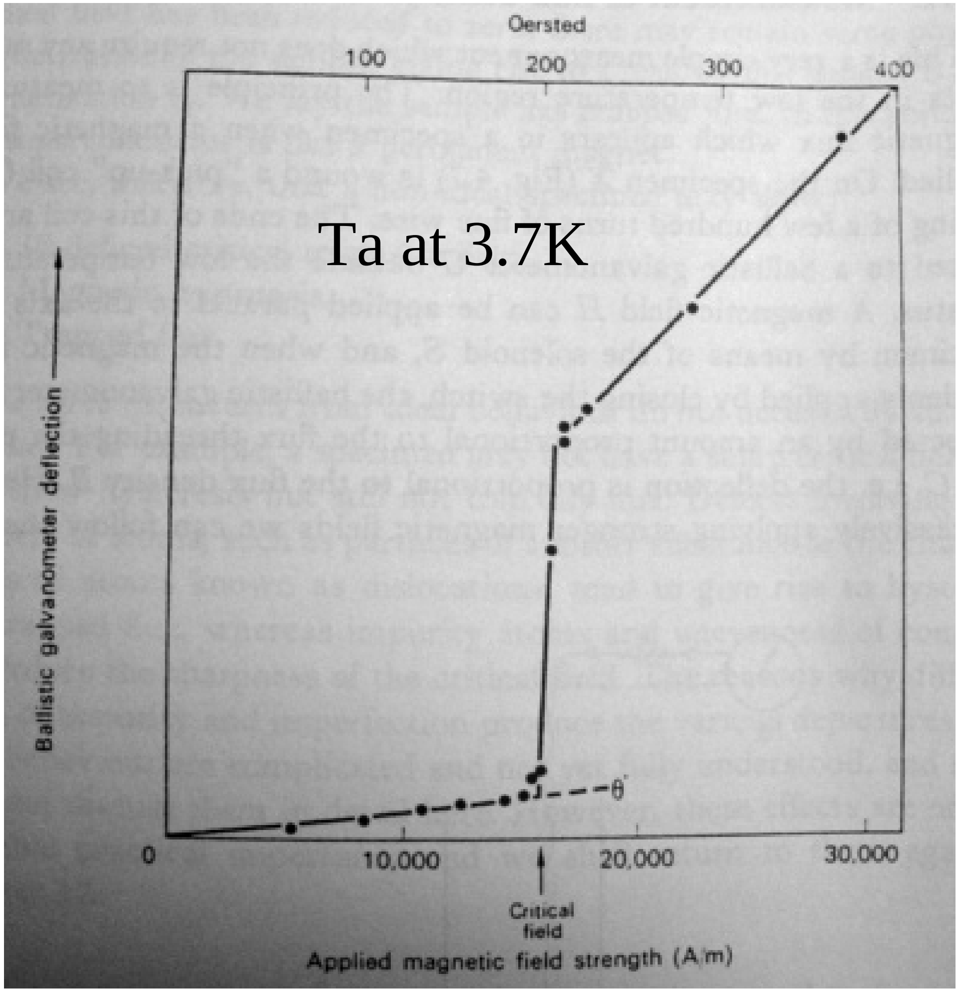
C: pick-up coil

S: solenoid to apply the field

When the switch is closed, the ballistic galvanometer is deflected. The deflection of G is proportional to *B*.



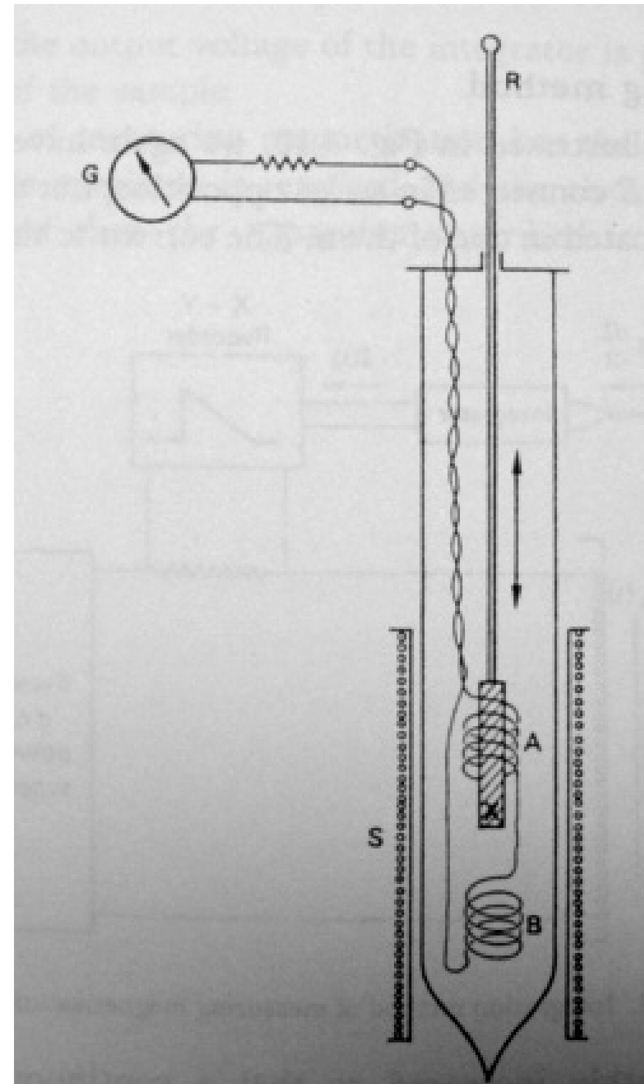
Ta at 3.7K


Oersted
100
200
300
400
Ballistic galvanometer deflection
θ'
0
10,000
20,000
30,000
Critical field
Applied magnetic field strength (A/m)

Measurement of magnetization VSM

The specimen is moved to and fro between pick-up coils A and B.

The galvanometer swings with an amount proportional to the magnetization *M*.



Integrating method

$$\mathcal{E} = \mathcal{E}_A - \mathcal{E}_B$$

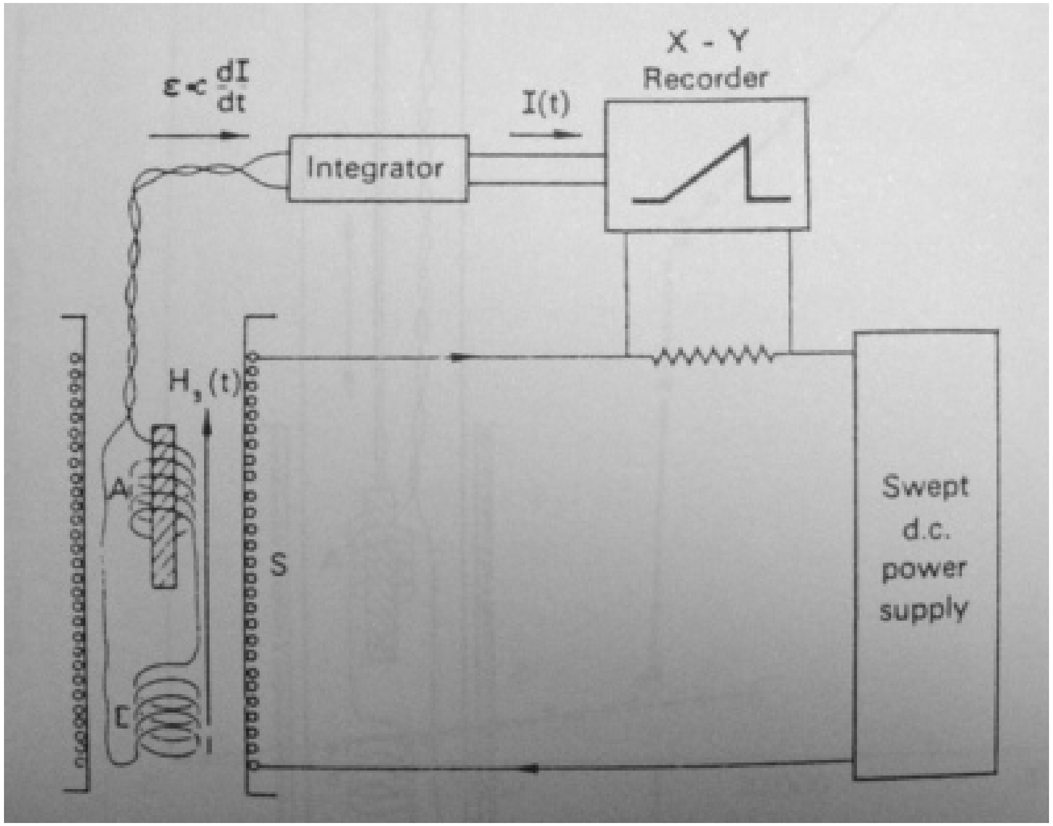
$$\propto \frac{d}{dt}\mu_0\left(H+M\right) - \frac{d}{dt}\mu_0 H$$

$$\mathcal{E} \propto \frac{dM}{dt}$$

Integrator:

$$V_{out} \propto \int V_{in}dt \propto M$$

cf: AC susceptometer



Homework

1. Use the definition of Gibbs free energy $G = U - TS + PV - \mu_0 H_a M$ to show the Gibbs free energy change due to a magnetic field H_a is given by.

$$\Delta g\left(H_a\right) = -\mu_0 \int_0^{H_a} M dH_a$$

5. Thermodynamics

Entropy

From $$\mu_0 \frac{H_C^2}{2} = g_n(T,0) - g_s(T,0)$$

We have $$\frac{\mu_0}{2}\left(H_C^2 - H_a^2\right) = g_n(T,0) - g_s(T,H_a)$$

$$G = U - TS + PV - \mu_0 H_a M$$

$$dG = dU - d(TS) + d(PV) - \mu_0 d(H_a M)$$
$$= -SdT + VdP - \mu_0 M dH_a$$

entropy $$s = -\left(\frac{\partial g}{\partial T}\right)_{p,H_a}$$

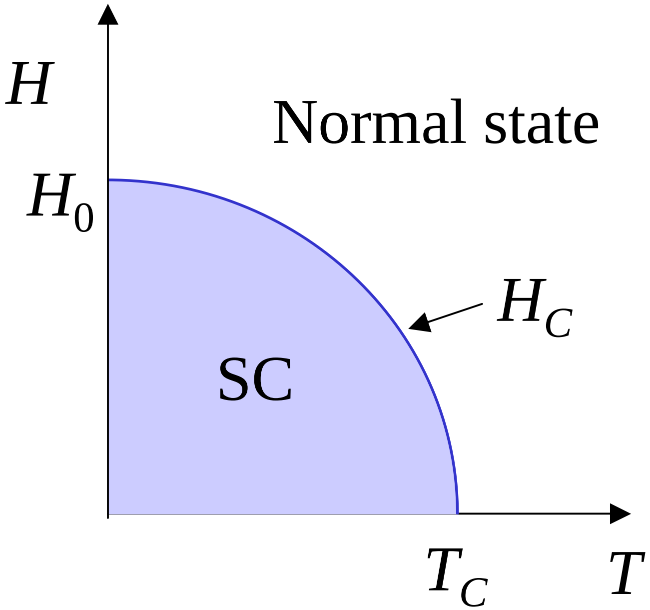
H_C is temperature dependent

thus $$s_n - s_s = \mu_0 H_C \frac{dH_C}{dT}$$

since $$\frac{dH_C}{dT} < 0$$

$$s_n > s_s$$

The superconducting state is more "order" than the normal state



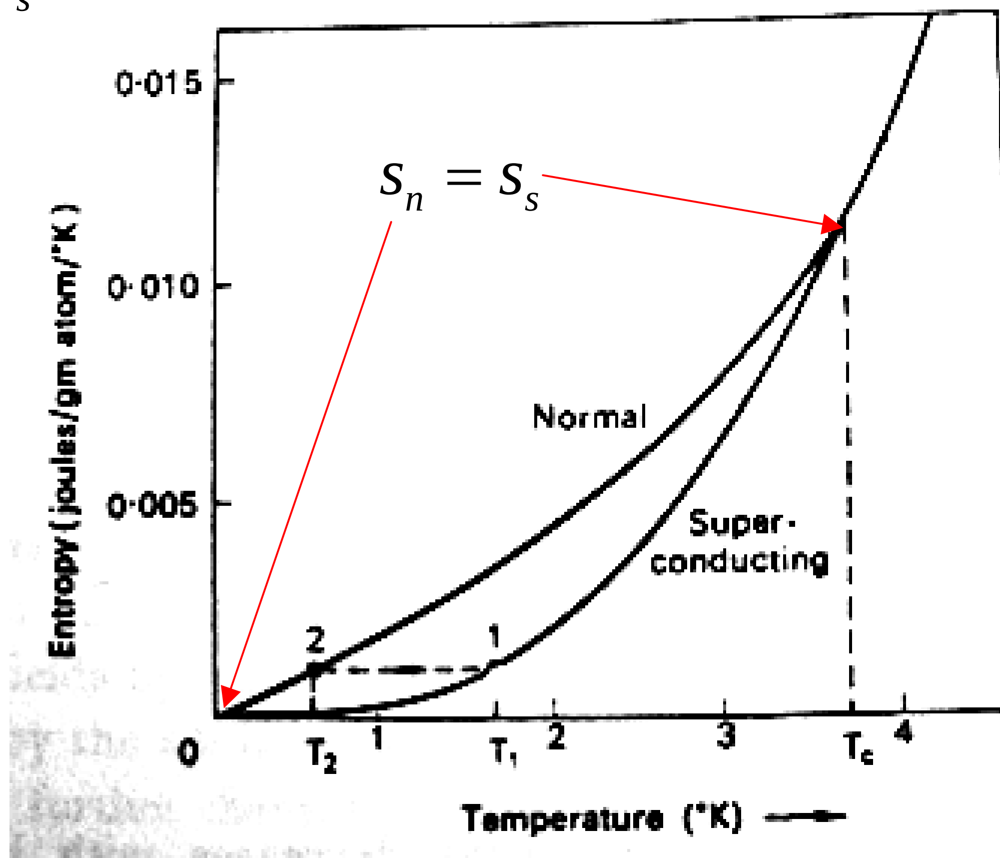
At T_C , H_C=0 $s_n = s_s$

According to the third law of thermodynamics,

$$s_n = s_s \qquad \text{at } T=0$$

Since H_0 is finite, this requires that

$$\frac{dH_C}{dT} = 0 \qquad \text{at } T=0$$



Entropy vs temperature

2^{nd} order phase transitions

At T_C $\quad s_n = s_s$

$$\left(\frac{\partial g}{\partial T}\right)_n = \left(\frac{\partial g}{\partial T}\right)_s$$

Both g and its derivative $\dfrac{\partial g}{\partial T}$ are continuous

A 2nd order transition

Features: no latent heat $\quad L = vT\left(s_s - s_n\right)$

a jump in specific heat $\quad C = vT\dfrac{\partial s}{\partial T}$

Specific heat

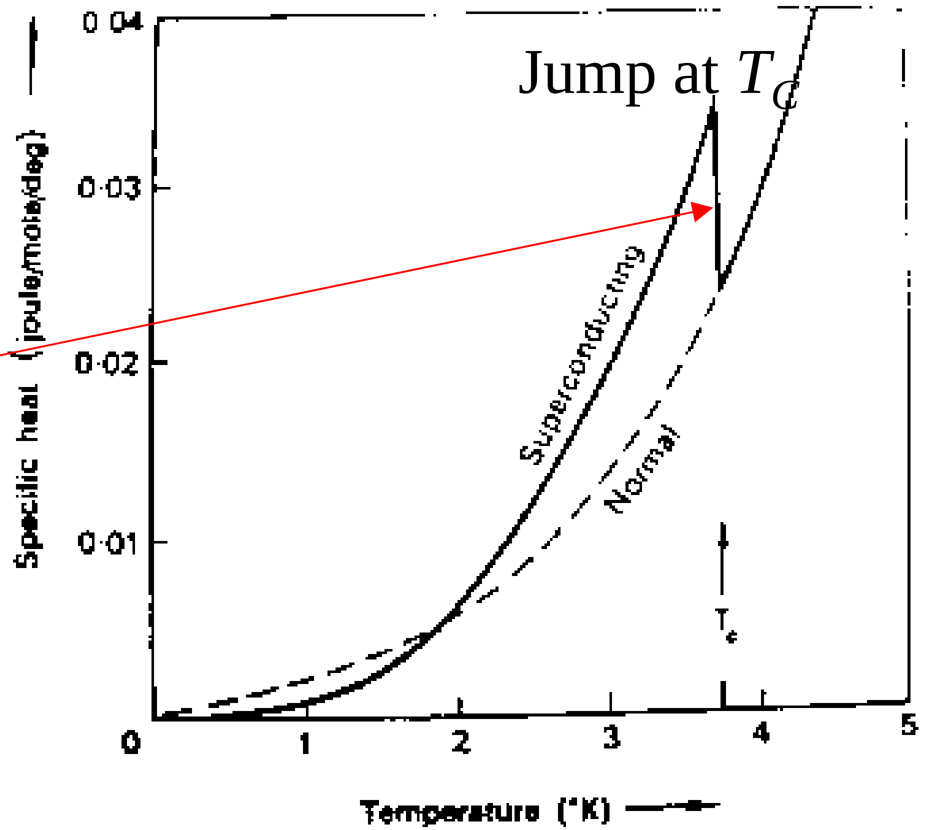
$$C_s - C_n = vT\frac{\partial(s_s - s_n)}{\partial T} = vT\mu_0\left[H_C\frac{d^2H_C}{dT} + \left(\frac{dH_C}{dT}\right)^2\right]$$

At T_C , H_C=0

$$(C_s - C_n)_{T_C} = vT_C\mu_0\left(\frac{dH_C}{dT}\right)^2_{T_C}$$

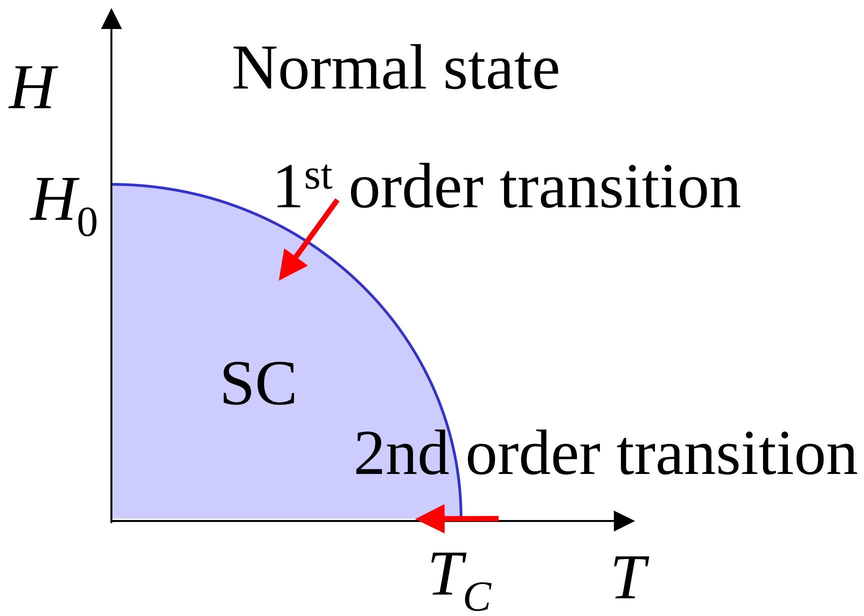
Rutger’s formula

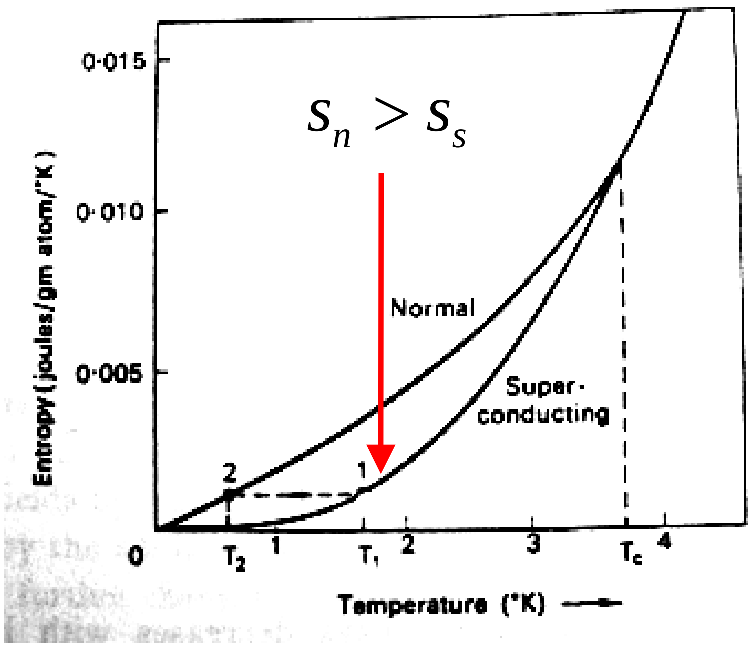
Jump at T_C



Latent heat at non-zero field

$$L = vT\left(s_s - s_n\right) = -vT\mu_0 H_C \frac{dH_C}{dT}$$





Adiabatic magnetization

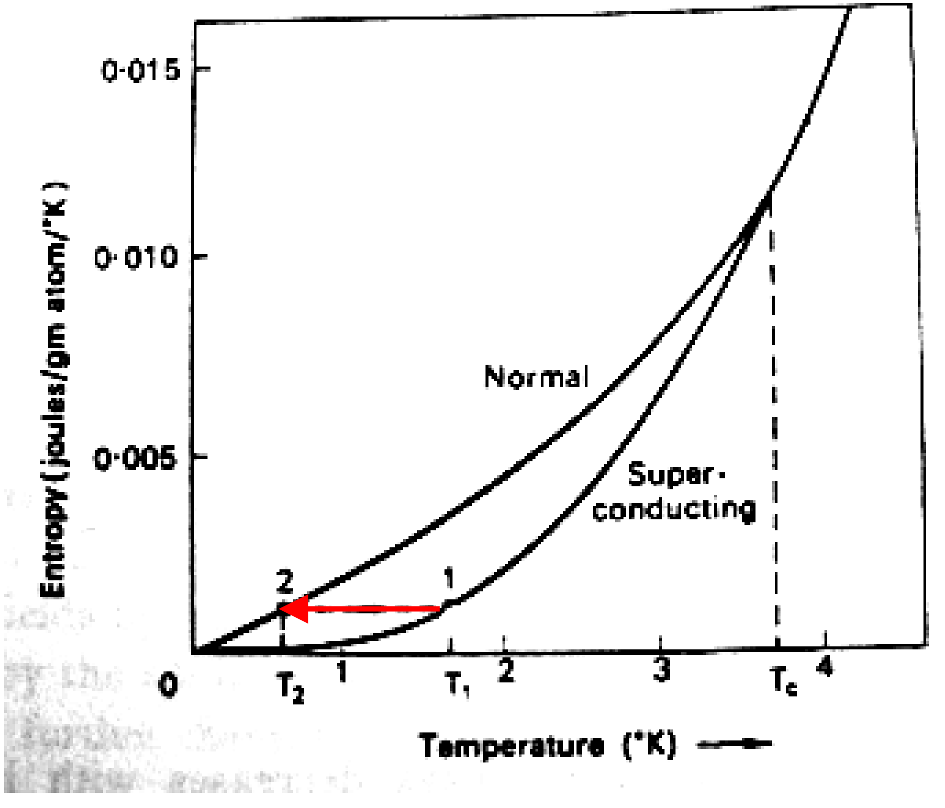
Adiabatic process, *s*=constant

$$dQ = TdS = 0$$

Adiabatically destructing the superconductivity by applying a large field can lower the temperature

Adiabatic demagnetization

Is used for ordinary magnetic materials in which the entropy decreases with application of field.



Specific heat

The specific heat of a metal is contributed from the lattice and conduction electrons

$$C = C_{latt} + C_{el}$$

The properties of the lattice do not change at the transition

$$C_s - C_n = \left(C_{el}\right)_s - \left(C_{el}\right)_n$$

For a normal metal

$$C_n = C_{latt} + \left(C_{el}\right)_n = A\left(\frac{T}{\theta}\right)^3 + \gamma T$$

θ: Debye temperature γ: Sommerfeld constant

At $T \sim T_C$

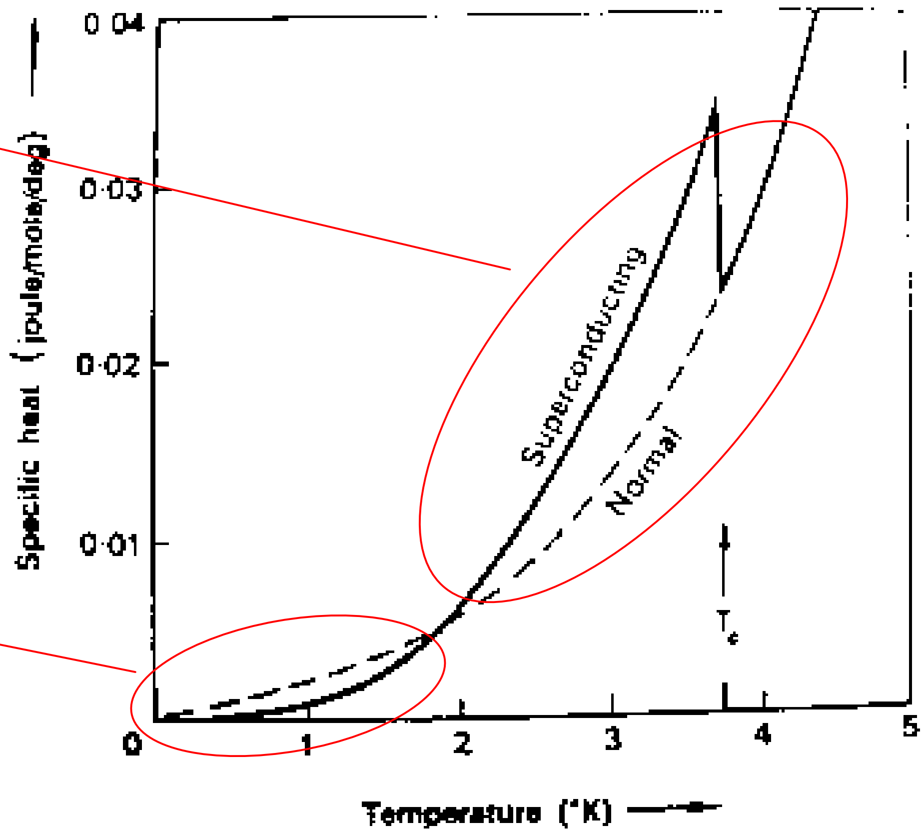
$$C_s > C_n$$

because $s_n > s_s$

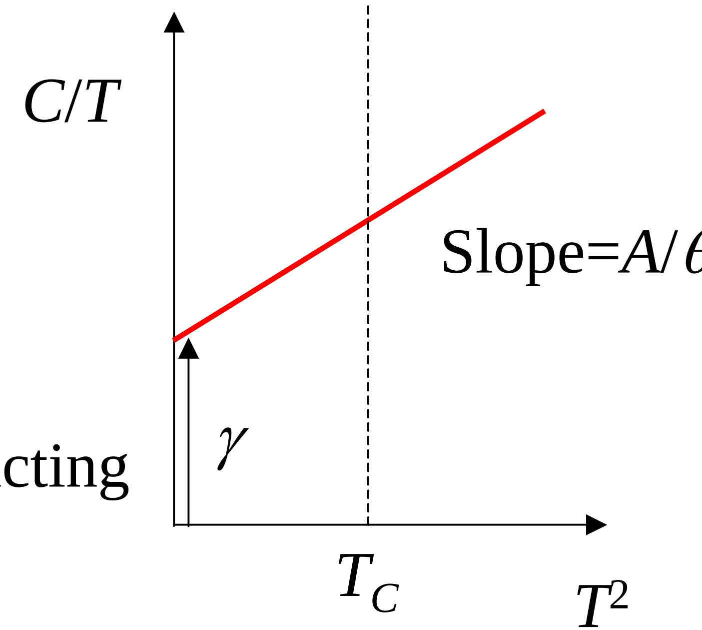
$$\frac{dS_s}{dT} > \frac{dS_n}{dT}$$

At low temperature

$$C_s < C_n$$



$$\frac{C_n}{T} = \frac{A}{\theta^3} T^2 + \gamma$$



At $T<T_C$, one can measure C for normal state ($H_a>H_C$)

The electronic part for superconducting state can be determined

$$\left(C_{el}\right)_s = ae^{-b/kT}$$

The form of the thermal activation : hint of a energy gap b/e

As T approaches T_C, b rapidly decreases to zero

Irreversible processes

Thermal conduction

The thermal conduction in a metal is mostly contributed by the conduction electrons

Superelectrons have negligible interaction with the lattice

The thermal conductivity in superconducting state is much smaller than that in normal state.

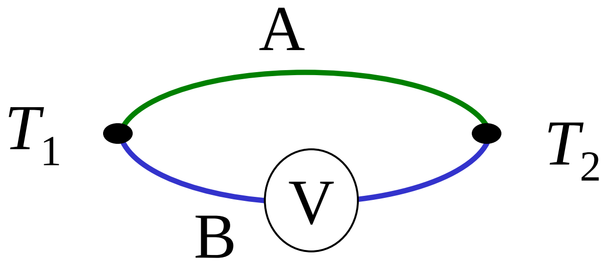
Thermoelectric effects

Due to zero resistance of the superconductors, the thermal e.m.f. should be zero. Therefore, Peltier and Thomson coefficients are zero.

In fact, thermoelectric effects may appear in type-II superconductors

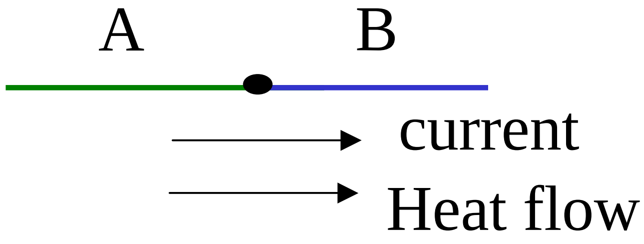
Seebeck(thermoelectric) effect

e.m.f. due to temperature gradient under conditions of zero electric current



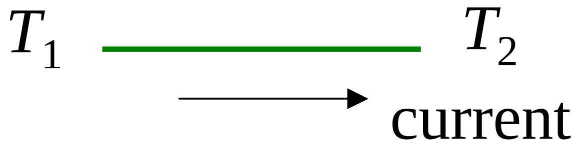
Peltier effect

Heat flow following an electric current across an isothermal junction



Thomson effect

Electric current traverses a temperature gradient



Homework

1. Apply Sommerfeld’s theory to show that the electronic specific heat of the metals is proportional to the temperature, i. e.

$$C_{el} = \gamma T$$

in which the constant γ is proportional to the density of states at Fermi surface.

2. Do the same calculation by assuming an energy gap Δ forming at the Fermi surface and show that

$$C_{el} = ae^{-e\Delta/kT}$$

at low temperature, $kT \ll \Delta$