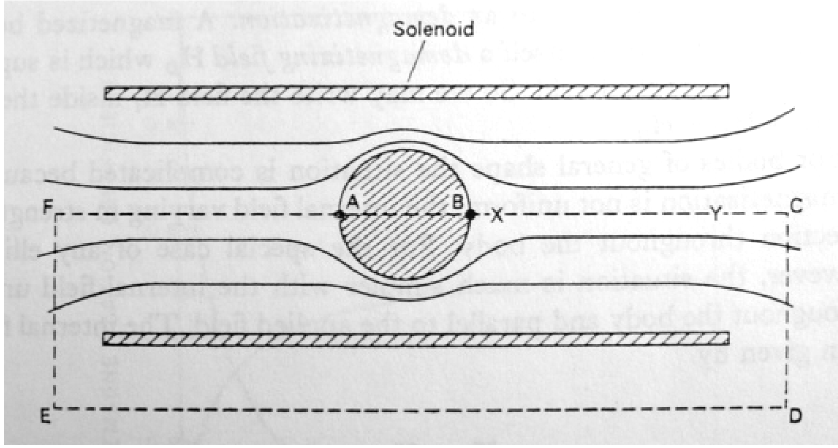
# 6. Intermediate state

# Demagnetization

Without the specimen

$$\oint \mathbf{H}\cdot d\mathbf{l} = \int_{AB} \mathbf{H}_a \cdot d\mathbf{l} + \int_{BCDEFA} \mathbf{H}'_{ext} \cdot d\mathbf{l} = Ni$$




With the specimen

$$\oint \mathbf{H}\cdot d\mathbf{l} = \int_{AB} \mathbf{H}_{in} \cdot d\mathbf{l} + \int_{BCDEFA} \mathbf{H}_{ext} \cdot d\mathbf{l} = Ni$$

Due to the diamagnetism of the specimen, $\mathbf{H}'_{ext} > \mathbf{H}_{ext}$

$(\mathbf{B}'_{ext} > \mathbf{B}_{ext})$

Therefore, $\mathbf{H}_a < \mathbf{H}_{in}$

For a paramagnetic specimen, we have $\mathbf{H}'_{ext} < \mathbf{H}_{ext}$ and $\mathbf{H}_a > \mathbf{H}_{in}$

⇨ The magnetic field is reduced, called demagnetization

# Demagnetizing factor

$$\mathbf{H}_{in} = \mathbf{H}_a - \mathbf{H}_D$$

Demagnetizing field

The demagnetizing field is not uniform for specimens of general shape

For a ellipsoid, the demagnetizing field is uniform and is parallel to the external field

$$\mathbf{H}_{in} = \mathbf{H}_a - n\mathbf{M}$$

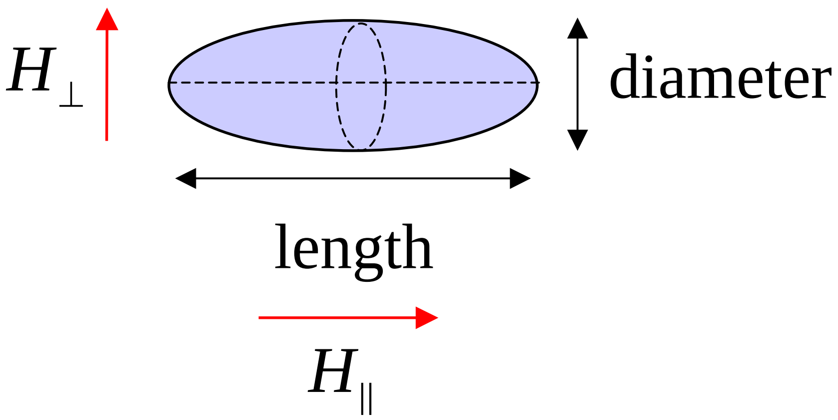
$n$ is called the demagnetizing factor

For a superconductor

$$\mathbf{M} = -\mathbf{H}_{in}$$

$$\mathbf{H}_{in} = \mathbf{H}_a + n\mathbf{H}_{in}$$

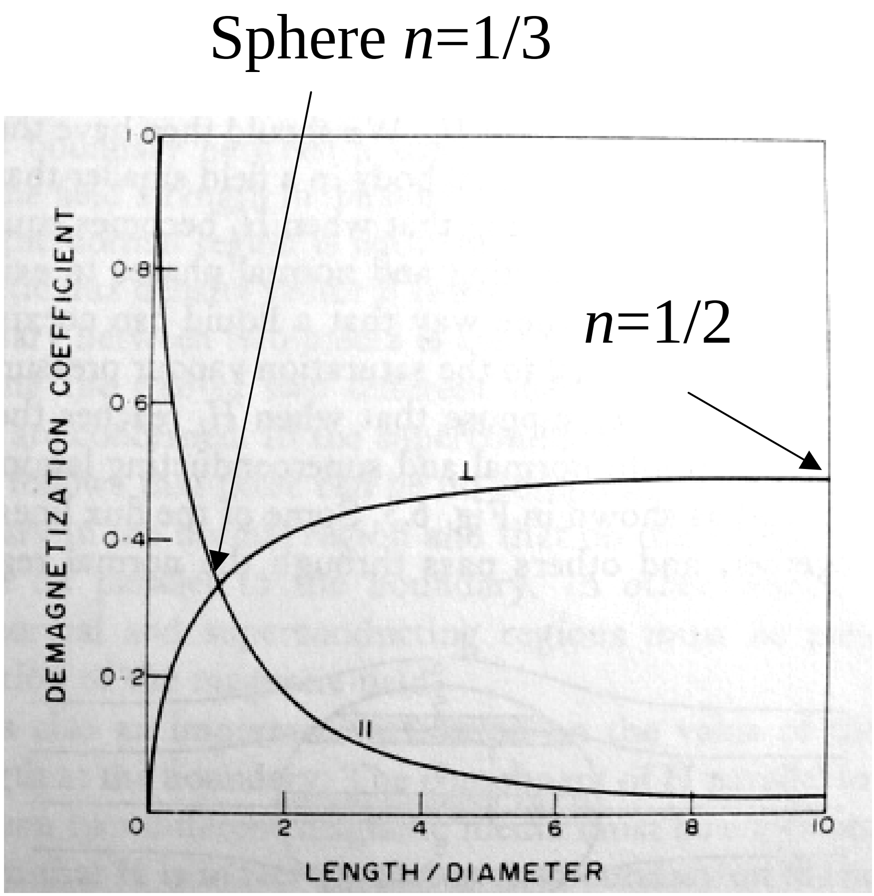
$$\mathbf{H}_{in} = \left(\frac{1}{1-n}\right)\mathbf{H}_a$$




For a long thin wire

$H_\parallel$ $n=0$

$H_\perp$ $n=1/2$



# Magnetic transition

In superconducting state

$$\mathbf{H}_{in} = \left(\frac{1}{1-n}\right)\mathbf{H}_a$$

$$H_a > (1-n) H_C$$

$H_{in} > H_C$ $\Longrightarrow$ The transition should occur

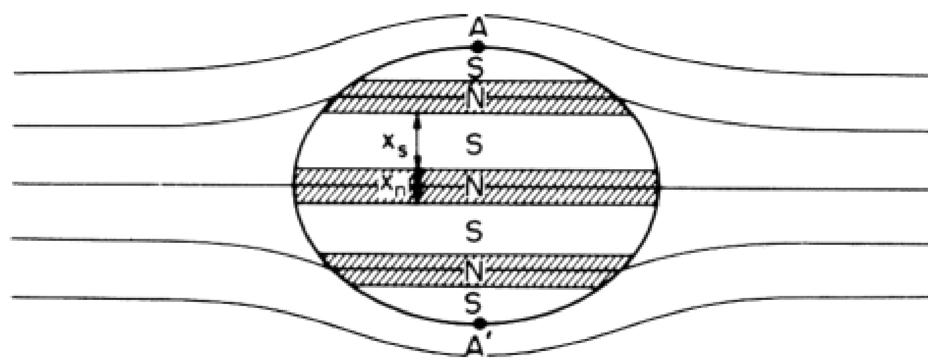
In normal state $M = 0$

$H_{in} = H_a < H_C$ $\Longrightarrow$ The normal state is not stable

The coexisting of superconducting and normal states is formed for $(1-n) H_C < H_a < H_C$

$\Longrightarrow$

called **intermediate state**



# NS Boundary

Some restrictions for a stationary boundary between a superconducting and a normal region

From the electrodynamics, we have

$$H_{\parallel}^{(n)} = H_{\parallel}^{(s)}$$

$$B_{\perp}^{(n)} = B_{\perp}^{(s)}$$

$$\Rightarrow \quad B_{\perp}^{(n)} = B_{\perp}^{(s)} = 0$$

$$H_{\parallel}^{(n)} = H_{\parallel}^{(s)} = H_C$$

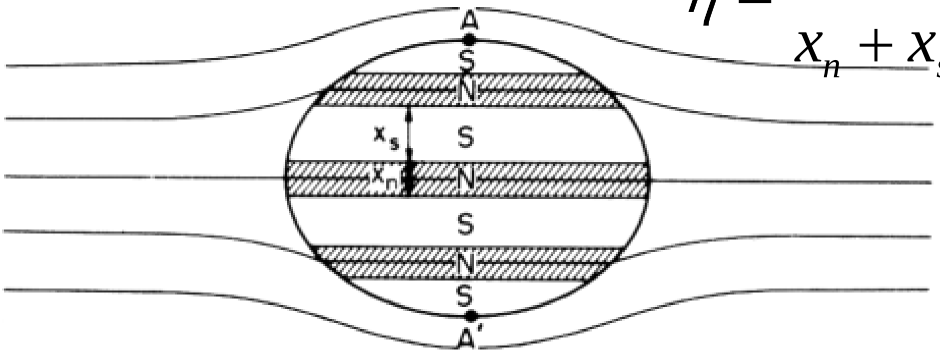
| N | S |
|---|---|
| $B^{(n)} = \mu_0 H^{(n)}$ | $B^{(s)} = 0$ |
| $H^{(n)} \geq H_C$ | $H^{(s)} \leq H_C$ |

The boundary should be parallel to the external field

# Magnetic properties

$$\eta = \frac{x_n}{x_n + x_s}$$

The average flux density in the ellipsoid body




$$\overline{B} = \eta B_n$$

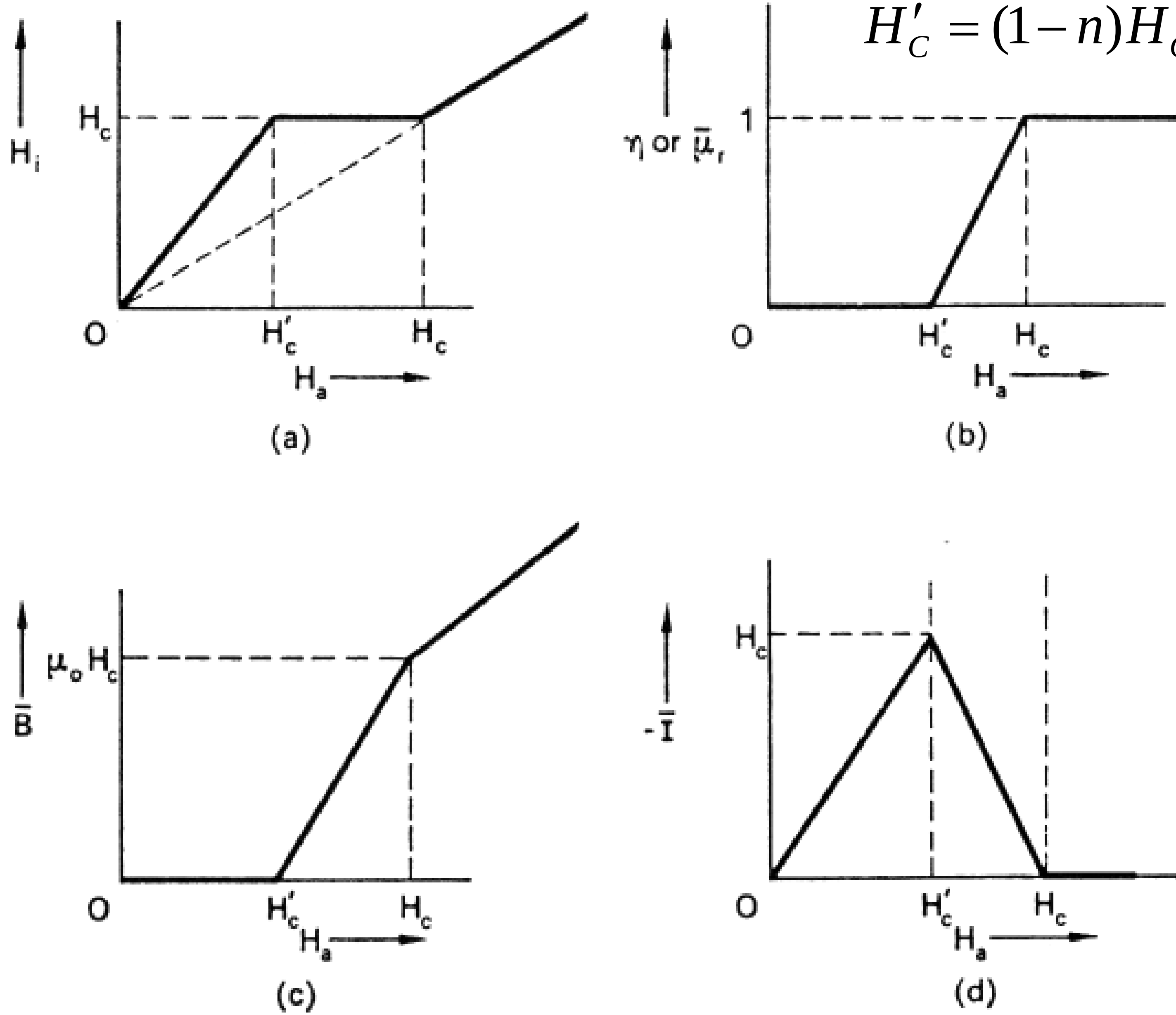
$$B_n = \mu_0 H_{in} \qquad \overline{B} = \eta \mu_0 H_{in}$$

average magnetization $\quad \overline{M} = \frac{\overline{B}}{\mu_0} - H_{in} \qquad \overline{B} = \mu_0 \left( H_{in} - \overline{M} \right)$

$$H_{in} = H_a - nM = H_a - n\left( \frac{\overline{B}}{\mu_0} - H_{in} \right) = H_a - n(\eta - 1) H_{in}$$

$$H_{in} = H_a \Big/ \left[ 1 - n(1-\eta) \right] = H_C$$

$$H'_C = (1-n)H_C$$

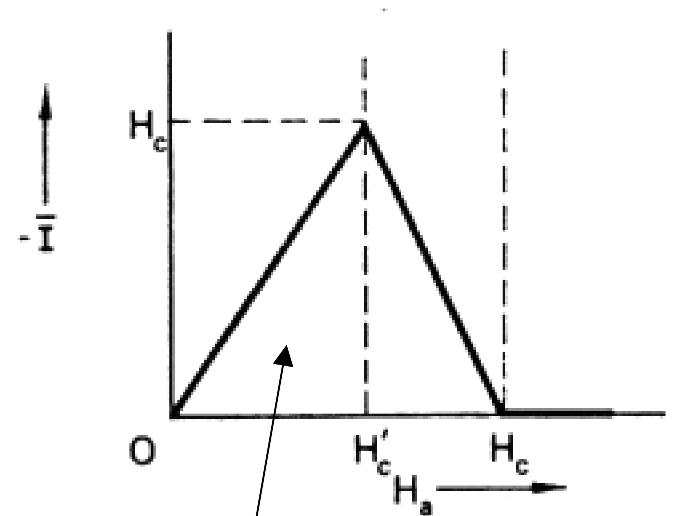


H_i
H_c
O
H'_c
H_c
H_a
(a)
η or μ̄_r
1
O
H'_c
H_c
H_a
(b)
B̄
μ_o H_c
O
H'_c
H_c
H_a
(c)
-Ī
H_c
O
H'_c
H_c
H_a
(d)

# Gibbs free energy

$$dG = -\mu_0 M dH_a$$

$$G(H_a) = G(0) - \mu_0 \int_0^{H_a} M dH_a$$




(I) $0 < H_a < (1-n) H_C$

$$M = -H_a$$

$$G(H_a) = Vg_s(0) + \frac{V\mu_0 H_a^2}{2(1-n)}$$

(II) $(1-n) H_C < H_a < H_C$

$$M = (\eta - 1) H_{in} = (\eta - 1) H_C$$

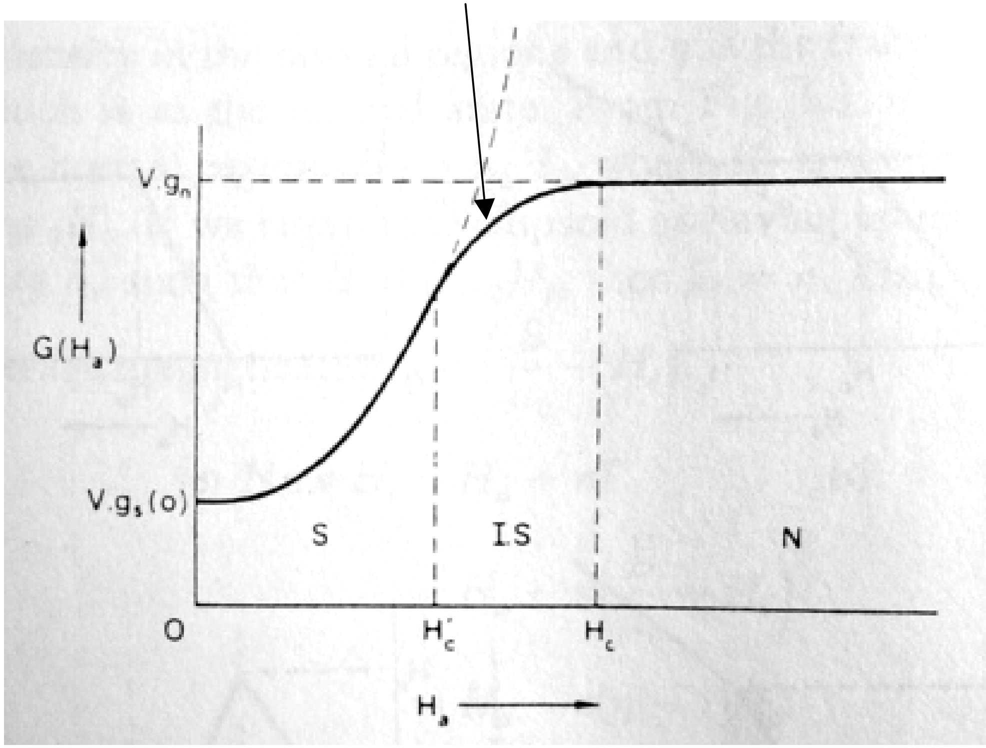
The area

$$G(H_a) = Vg_s(0) + \frac{V\mu_0 H_C}{2n}\left[ H_a\left(2 - \frac{H_a}{H_C}\right) - H_C(1-n)\right]$$

(III) $H_C < H_a$

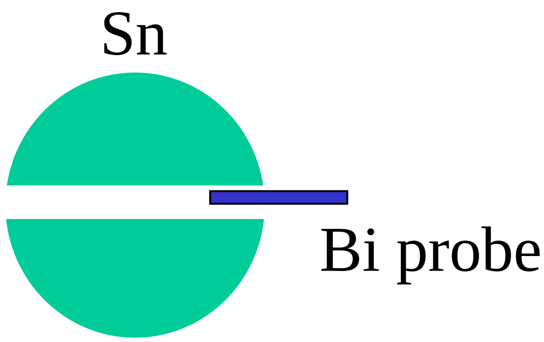
$$G(H_a) = Vg_s(0) + \frac{V\mu_0 H_C^2}{2} = Vg_n$$

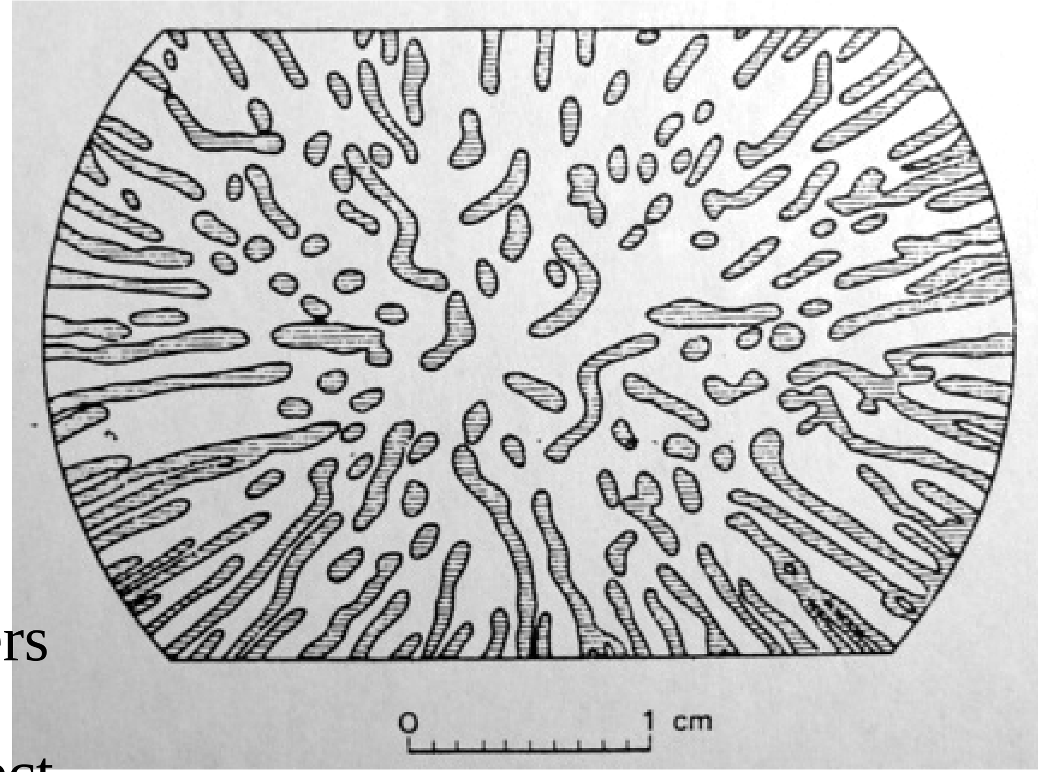
Lower free energy



# Experimental observations

1. MR sensor




2.Use ferromagnetic powders

3.Use superconducting powders

4.Polarized light: Faraday effect

For a thin film: $n = 1 - t/2a$ $\qquad H'_C \sim 0$

# The Domain

$$\eta = \frac{x_n}{x_n + x_s}$$

Can be determined

The size of the domain?

The NS boundary accompanies with a surface energy

The energy is proportional to the boundary area

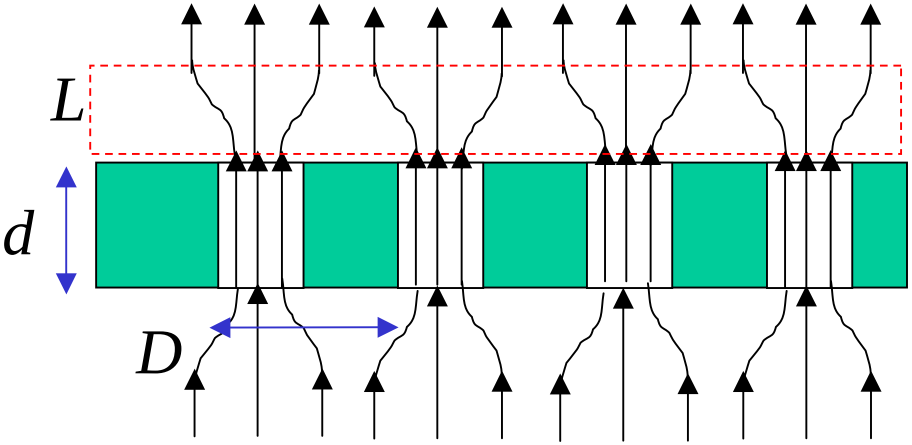
Positive surface energy → The area is as small as possible

Negative surface energy → No Meissner effect

# Intermediate state in a flat slab

Landau's work (1937)

Periodic laminar structure




surface energy per area

$$a = \frac{1}{2}\mu_0 H_C^2 \Delta$$

The free energy: $F_1$ and $F_2$

Surface energy

Non-uniform field outside the slab

$$F_1 = \frac{2da}{D} = \frac{2d\Delta}{D}\left(\frac{1}{2}\mu_0 H_C^2\right)$$

Surface energy per unit area of the slab

The non-uniform field region extend to a distance, $L$~min($x_n$,$x_s$)

$$L \simeq \frac{1}{x_n^{-1} + x_s^{-1}} = \eta(1-\eta)D \qquad \eta = \frac{x_n}{x_n + x_s}$$

$$\frac{1}{2}\eta\mu_0 H_n^2$$ Field energy density of the intermediate state

$$\frac{1}{2}\mu_0 H_a^2 = \frac{1}{2}\mu_0\left(\eta H_n\right)^2$$ energy density of the uniform field

$$\Rightarrow \quad F_2 = \frac{1}{2}\mu_0\left(\eta-\eta^2\right)H_n^2 L = \frac{1}{2}\mu_0\eta^2\left(1-\eta\right)^2 DH_n^2$$

Total free energy

$$F_1 + F_2 = \frac{1}{2}\mu_0\left[\frac{2d\Delta}{D}H_C^2 + \eta^2\left(1-\eta\right)^2 DH_n^2\right]$$

Minimize $F$ with respect to $D$

$$D^2 = \frac{2d\Delta H_C^2}{\eta^2\left(1-\eta\right)^2 H_n^2}$$

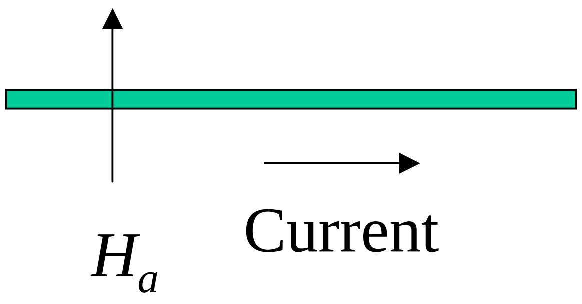
$$D \approx \frac{\sqrt{d\Delta}H_C}{\eta\left(1-\eta\right)H_n}$$

A refined calculation yields

$$\frac{H_C}{\eta\left(1-\eta\right)H_n} = \frac{H_C}{\left(1-\eta\right)H_a}$$

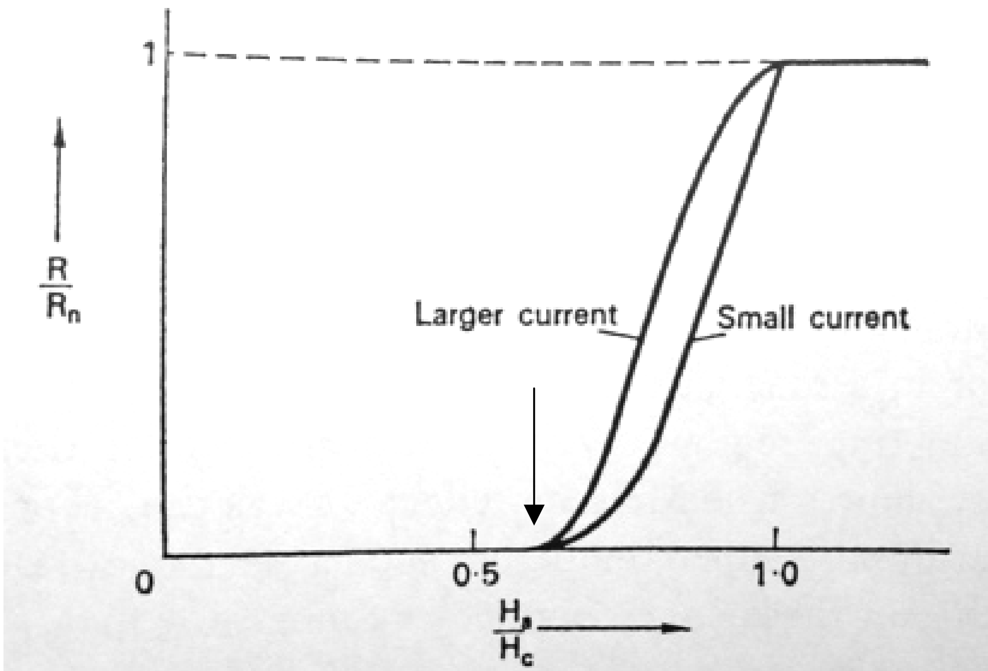
$$\frac{x_s}{d} = \frac{D\left(1-\eta\right)}{d} \sim \frac{\sqrt{\Delta/d}}{H_a/H_C}$$

# Restoration of resistance




$H_\perp$ $n$=1/2

$$H_C' = 0.5H_C$$




Because of a positive surface energy

$$H_C' > 0.5H_C$$

# Surface energy

Coherence: proposed by A. B. Pippard in 1957

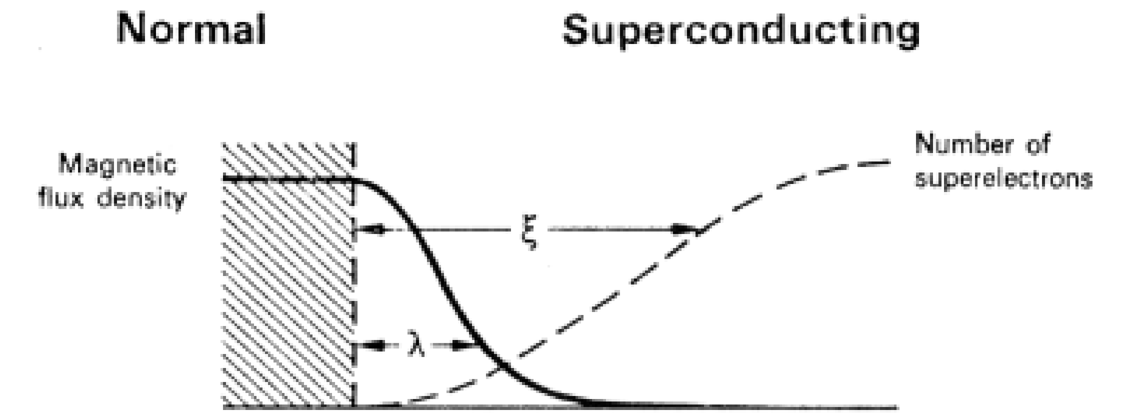
The number density of super electrons $n_s$, cannot change rapidly with position

Coherence length $\xi$ ~$10^{-4}$cm

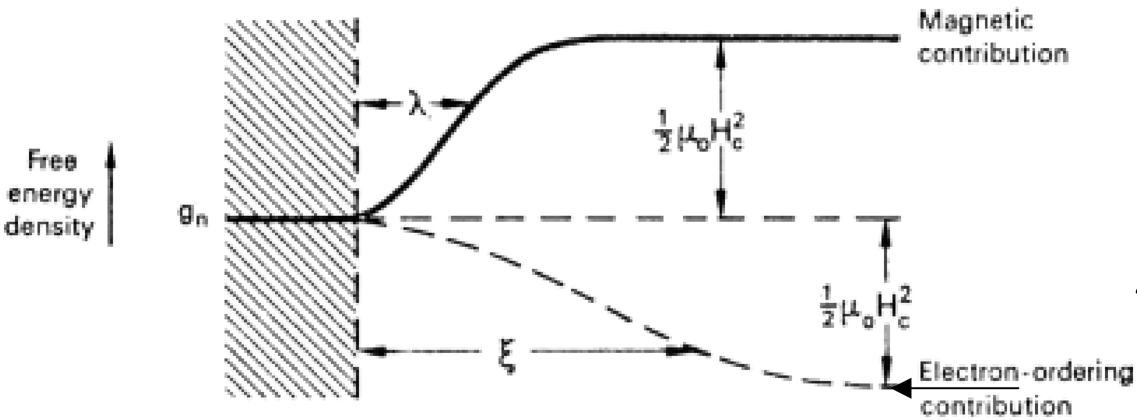
Impurity effect $\xi \sim \sqrt{\xi_0 l_e}$

A hint of the coherence: extreme sharpness of the transition

$\Delta T$ can be as small as $10^{-5}$ K



(a) Penetration depth and coherence range at boundary



(b) Contributions to free energy

$$\mu_0 \frac{H_C^2}{2} = g_n(T,0) - g_s(T,0)$$

$g_s(0)$

(c) Total free energy

For $\xi > \lambda$ the surface energy is positive

$$a = \frac{1}{2}\mu_0 H_C^2 \Delta \qquad \Delta \sim (\xi - \lambda)$$

For Pb or Sn, $\Delta \sim 5\times10^{-5}$ cm

$\lambda$~3.9x10$^{-6}$cm

⇒ $\Delta \simeq \xi \gg \lambda$

$\xi$ is temperature dependent and increases at higher temperature

At $T_C$, $\xi$ becomes infinite

# Homework

1. Show that the demagnetizing factor of a sphere is 1/3.

2. Evaluate the surface energy per area by assuming that the position dependence of the $n_s$ obeys the exponential law, i e. $n_s \sim n_s(0)\left(1-e^{-x/\xi}\right)$ , in which $x$ is the distance from the NS boundary and $n_s(0)$ is the number density deep inside the superconduting region.

# 7. Transport currents in superconductors

# Critical currents

$$\mathbf{J} = \mathbf{J}_i + \mathbf{J}_H$$

Transport current    Screening current

The transport current generates a magnetic field, which is screened by a surface current.

A superconducting wire loses its zero resistance when the total magnetic field strength exceeds the critical field strength at any point on the surface.

In the absence of external field

$$2\pi a H_i = i$$

⇨

$$2\pi a H_C = i_C$$

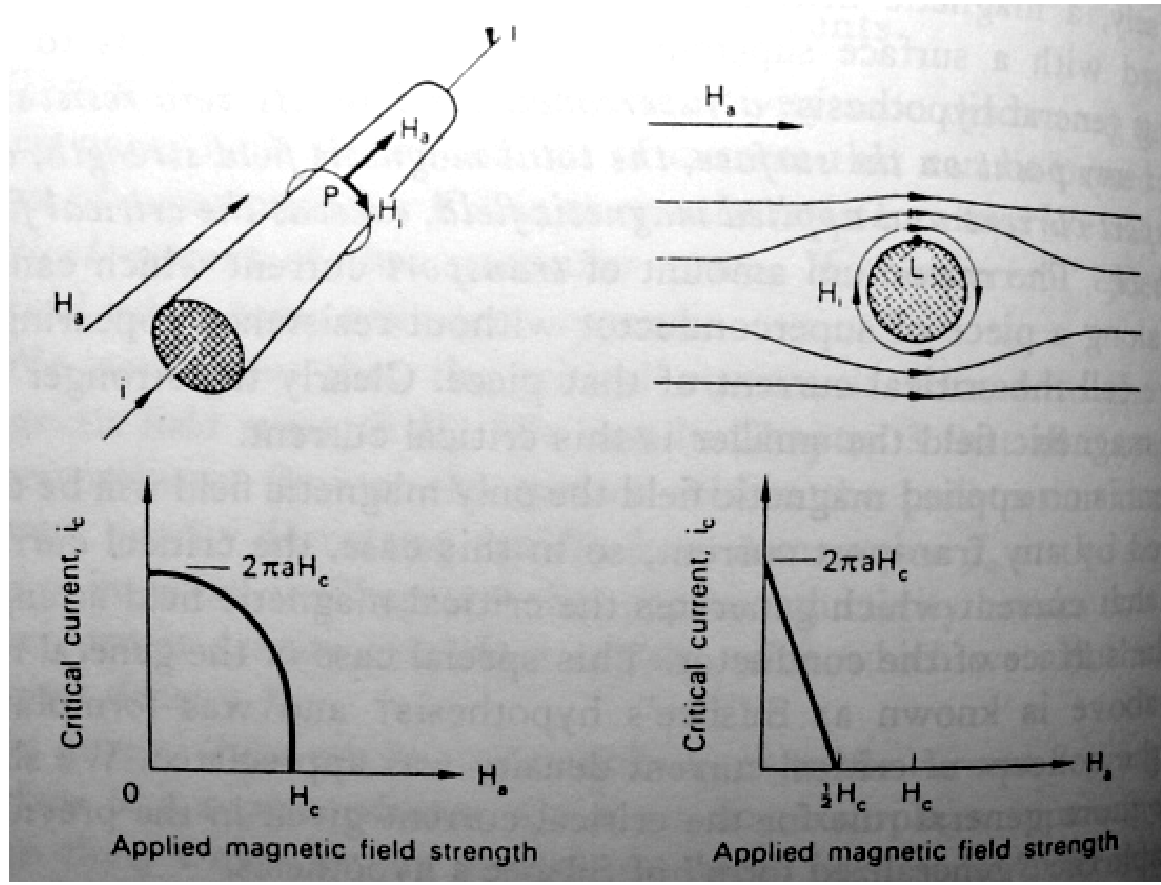
In a longitudinal field

$$H_C^2 = H_a^2 + \left(i_C / 2\pi a\right)^2$$

In a transverse field

$$H_C = \boxed{2} H_a + i_C / 2\pi a$$

Demagnetizing factor

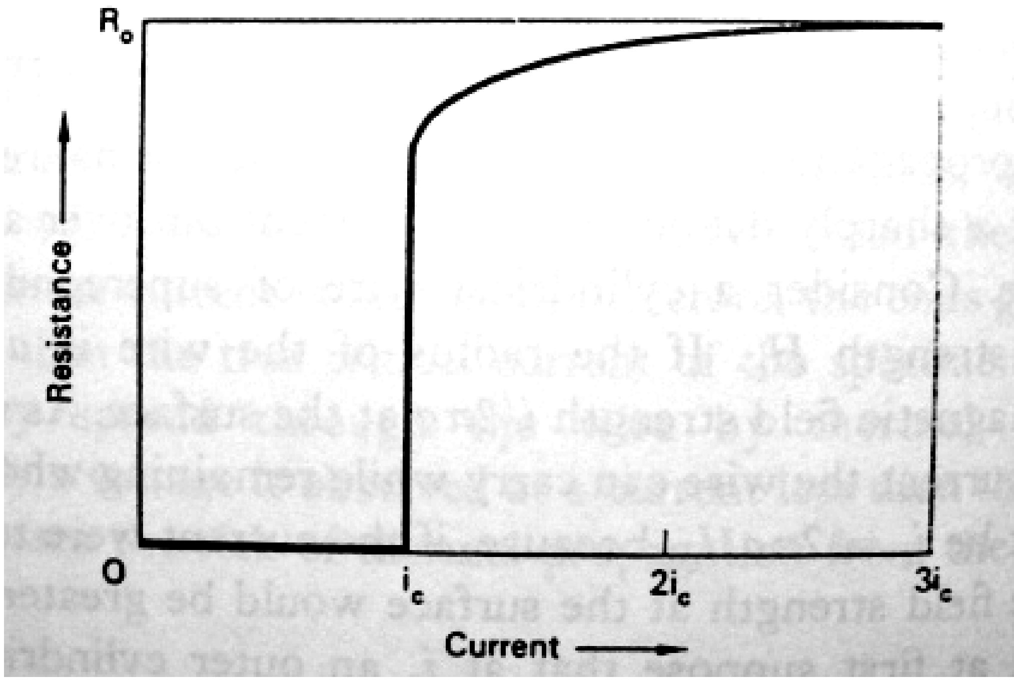



Longitudinal field transverse field

# Intermediate state induced by a current

Once the current exceeds $i_c$, the wire cannot have a superconducting cylindrical core.

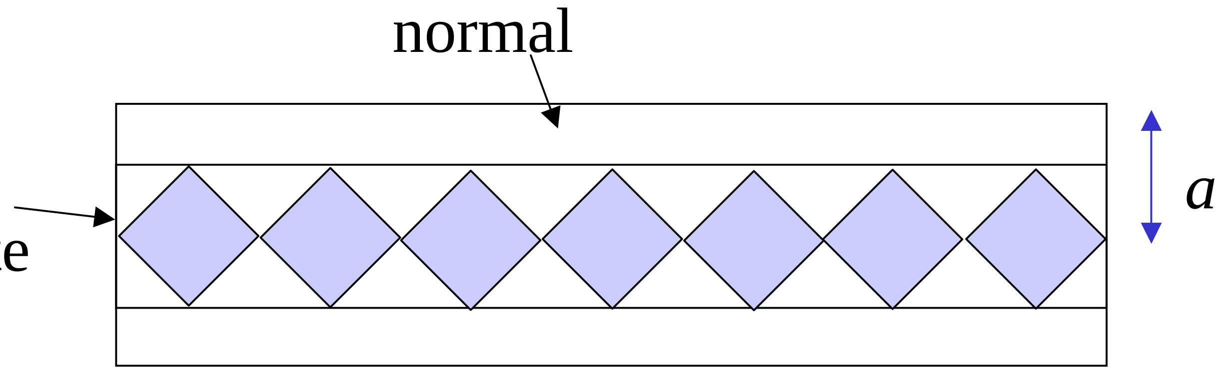
When the current is uniformly distributed, the field inside the superconducting wire becomes smaller than $H_C$




An intermediate state with a non-zero resistance

# London's model

F. London (1937)




In the intermediate state

$$H(r) = H_C$$

$$H(r) = \frac{i(r)}{2\pi r} \qquad i(r) = 2\pi r H_C$$

Current density

$$J(r) = \frac{1}{2\pi r}\frac{di}{dr} = \frac{H_C}{r}$$

From $\rho(r)J(r) = E$

$$\rho(r) \propto r$$

$$J(r) = \frac{Er_1}{\rho r}$$ Resistivity $\rho$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$ Electric field is constant of $r$

$$r_1 = \frac{H_C \rho}{E}$$

The total current in the core should generate $H_C$ at the core surface

$$J(r) = \frac{Er_1}{\rho r} \qquad i_1 = i(r_1) = 2\pi r_1 H_C = \frac{2\pi\rho H_C^2}{E}$$

The current in the outer shell is

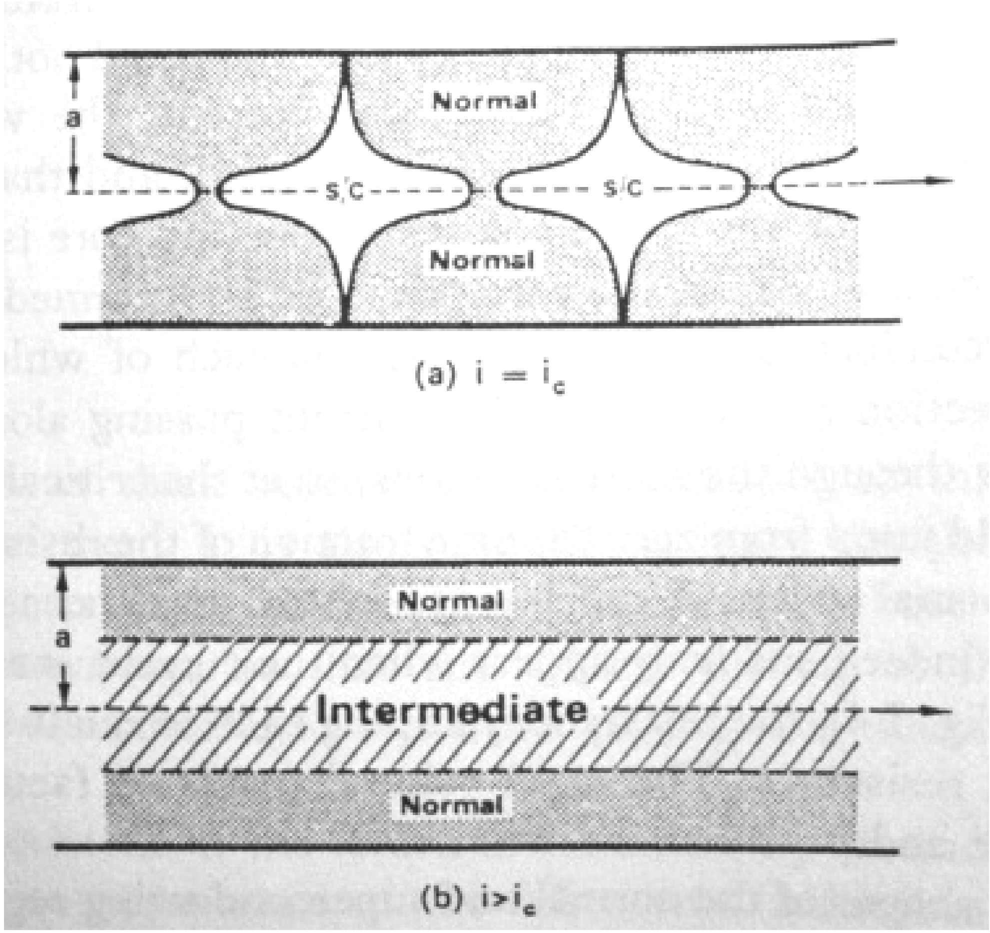
$$i_2 = \frac{E}{\rho}\pi\left(a^2 - r_1^2\right) = \frac{\pi a^2 E}{\rho} - \frac{\pi\rho H_C^2}{E}$$

$$i = i_1 + i_2 = \frac{\pi a^2 E}{\rho} + \frac{\pi\rho H_C^2}{E}$$

$$E = \frac{2\rho i}{\pi a^2}\left\{1 \pm \left[1 - \frac{i_C}{i}\right]^{1/2}\right\} \qquad i_C = 2\pi a H_C$$

$$\Longrightarrow \quad \frac{R}{R_n} = \frac{1}{2}\left\{1 \pm \left[1 - \frac{i_C}{i}\right]^{1/2}\right\}$$

For $i > i_C$




Suggested structure of the intermediate state