II. Size & Scale in Biology

Introduction to Biomechanics
2008/10/1

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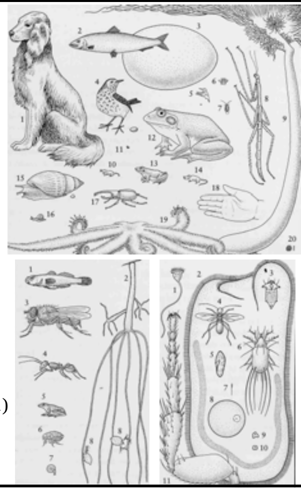

Size span in organisms—
Length: 10^8 –fold (0.3 μm – 30 m)
Mass: 10^{24}- fold

What would happen if they ate like the other?


Need fur 20 cm thick

Body temp > 100 °C

(by Max Kleiber)

Why?

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Lengths, Surface Areas, and Volumes

Surface area (S) vs. Volume (V) – one of the most important size relationship to biol. design

V	Energy required	Heat needed or produced	Waste produced
S	Materials acquired	Heat absorbed or dissipated	Materials excreted

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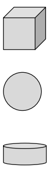
Relationship b/w S and V:



	S	V
Cubes	$6l^2$	l^3
Spheres	$4\pi r^2$	$\frac{4}{3}\pi r^3$
Cylinders	$4\pi r^2$	πr^3

$\propto L^2$ $\propto L^3$ ⇨ $\frac{S}{V} \propto L^{-1}$

Implications?

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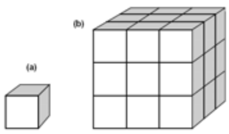
Isometry (same shape, different size), adj. isometric
↕ Allometry (adj. allometric)



$$\frac{L_b}{L_a} = (\frac{3}{1})^1 = 3$$

$$\frac{S_b}{S_a} = (\frac{3}{1})^2 = 9$$

$$\frac{V_b}{V_a} = (\frac{3}{1})^3 = 27$$

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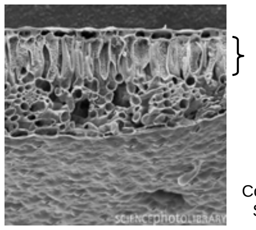
The solutions–

1. to change shape (allometry)

2. to increase the effective area

Ex. Tree leaves

Outer surface of a tree: 200 m^2

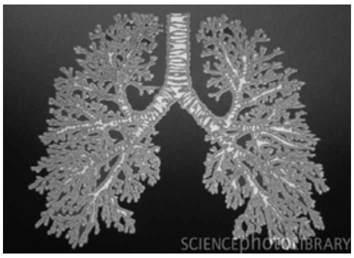


Colored SEM

Internal surface for gas exΔ: 30x

➔ Total surface: 6000 m^2

Ex. Lung



Lung ~ 6 liter

But 3 x 10^8 aveoli（肺氣泡）

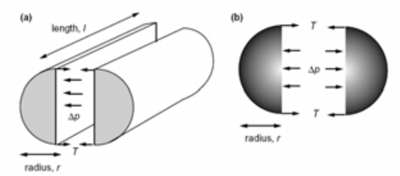
➔ Total surface: 50-100 m^2

Pressure, Tension, and Radius of Curvature

Experiences of frying a sausage?

Did the skin ever split? What's the pattern?

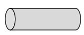
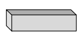
Splitting a cylinder lengthwise — Splitting a sphere *or* Splitting a cylinder hoop wise



Force from pressure	$\Delta p \cdot (2rl)$	$\Delta p \cdot (\pi r^2)$
Tension force	$T \cdot (2l)$	$T \cdot (2\pi r)$
T vs. *ΔP*	$T = \Delta pr$	$T = \frac{\Delta pr}{2}$

<u>Consequences on cylinders</u>:

To think about…

1. Tire walls vs. inflating pressures for bike (~ 700 kPa) and car (~200 kPa)
2. Why use  instead of  to transport water?
3. Xylem (20 μm thick) can withstand 2MPa pressure difference.
4. Capillaries have ~ 1/3 internal pressure of arteries, but only 1/2000 thick– why don't they explode?

We have...

talked about size in fairly loose terms

predicted relationships from underlying physical and geometrical rules

Now...

measurements → scale-dependence of design

→ the study of "**scaling**" or "**allometry**"

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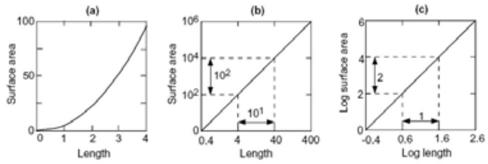
Scaling Empirically

S vs. L



$y = ax^b$

To get b: $\log y = \log a + b \log x$

Slope of log(y) vs. log(x)

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ON SIZE AND LIFE

THOMAS A. McMAHON AND JOHN TYLER BONNER

1983

Other books:

Schmidt-Nielsen (1984) (Animal physiologist)

Niklas (1994) (Botanist)

Pennycuick (1992) (*Newton Rules Biology;* Bird flight)

Brown & West (2000)

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Some Specific Scaling Factors ($y = ax^{\boldsymbol{b}}$)

x: a measure of organism's size, for animals, usually it's <u>body mass</u> (M)

y: variable of interest

If		
b = 0	y is size independent	
b = 0.33	$y \propto L$	Isometric
b = 0.67	$y \propto A$	Isometric
b = 1	$y \propto V$	Isometric

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Some real numbers on b (mammals)

Item	Scaling factor
Surface area	0.65
Skeletal mass (terrestrial)	1.08
Skeletal mass (cetaceans)	1.02
Muscle mass	1.00
Metabolic rate, minimal	0.75
Effective lung volume	1.03
Frequency of breathing	-0.26
Kidney mass	0.85
Liver mass	0.87
Brain mass	0.70

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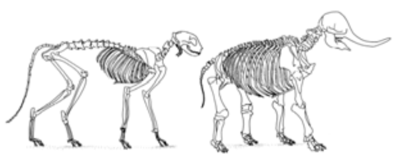
<u>Skeleton</u>

If considering the function...

Cross-section area (CSA) $\propto M^1$

and, if assuming length (L) $\propto M^{0.33}$

⇒ Skeleton mass = CSA × L $\propto M^{1.33}$



But empirical data: b = 1.08 < 1.33

Consequences & solutions

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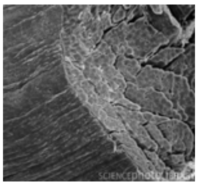
Muscle



Muscle force $\propto$ CSA

Assuming CSA$\propto M^{0.67}$

Work done by muscle = F $\times$ d

$\propto M^{0.67} \times M^{0.33} \propto M^1$

Consequences?

Small animals are stronger

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Respiration (Lung)

The fact:

Metabolic rate $\propto M^{0.75}$

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O2 supply = Eff. Lung Vol. $\times$ Breath. Freq. $\propto M^{0.77}$

Blood supply = Heart Vol. $\times$ Beat. Freq.

$\propto M^{(0.98-0.25)} = M^{0.73}$

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