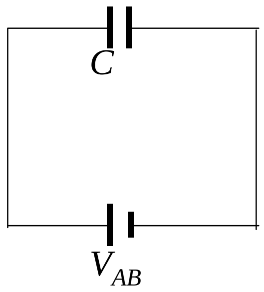
## 電容、電阻與電路(Capacitance, Resistance and Circuits)

- 電容與電能之儲存
- 電流與電阻
- 簡易電路
- 電容之充放電

**電容與電能之儲存**

**電容之定義與計算**

(1) **定義**

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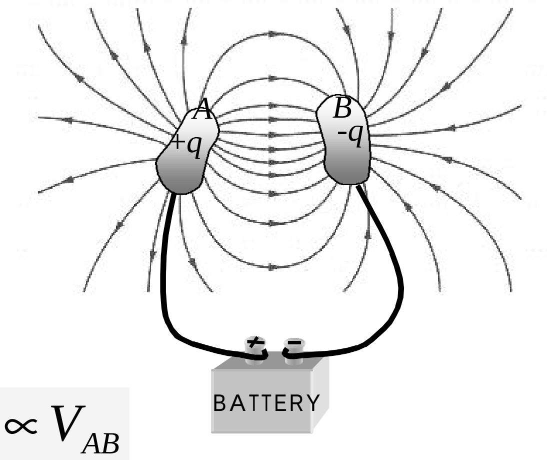
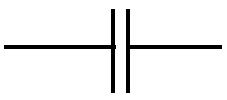
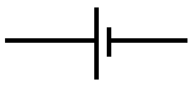


**電容器** (capacitor)



**電池** (battery)

$V_{AB}=V_A-V_B$

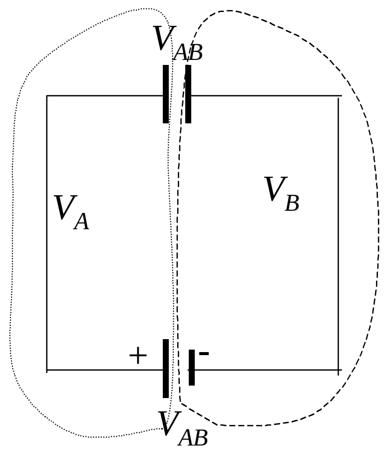
$$q \propto V_{AB}$$





**定義電容**(capacitance)

$$q = CV_{AB}, \qquad C \equiv \frac{q}{V_{AB}}$$



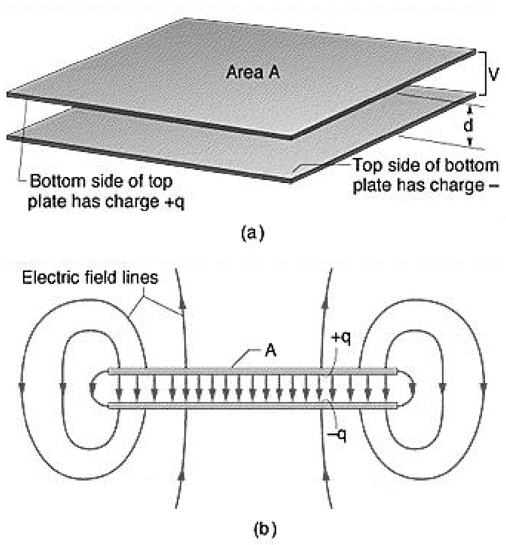

**單位** $\left[\frac{\mathrm{C}}{\mathrm{V}}\right] \equiv [farad] \equiv [\mathrm{F}]$ （法拉）

(2)**電容的計算**

**一般計算步驟：**

1. 令正負電極各帶電$+q$及$-q$；
2. 計算所產生之電場E(利用高斯定律)；
3. 計算正極與負極間之電位差$V$(路徑積分$-\int \mathbf{E}\cdot d\mathbf{s}$)；
4. $C=q/V$。

**例題** **平行導電板電容**(a parallel-plate capacitor)



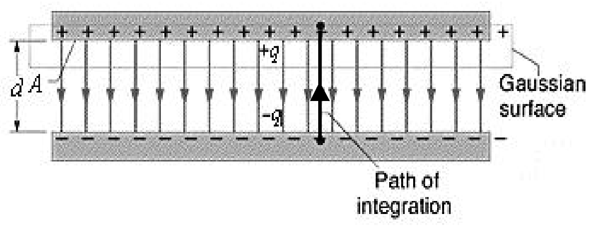

**假如導電板的寬度遠大於$d$，我們可以假設電場是均勻的，而忽略邊緣的效應。**

**由高斯定律**

$$e_0\oint \mathbf{E}\cdot d\mathbf{A}=q=e_0EA$$
$$E=\frac{q}{e_0A}$$

**由圖中所示之路徑由下電板積分到上電板表面可得**

$$V=-\int_0^d \mathbf{E}\cdot d\mathbf{s}=-\int_0^d(-Eds)=\int_0^d Eds$$
$$=\int_0^d \frac{q}{e_0A}ds=\frac{qd}{e_0A}$$
$$C=\frac{q}{V}=e_0\frac{A}{d}$$







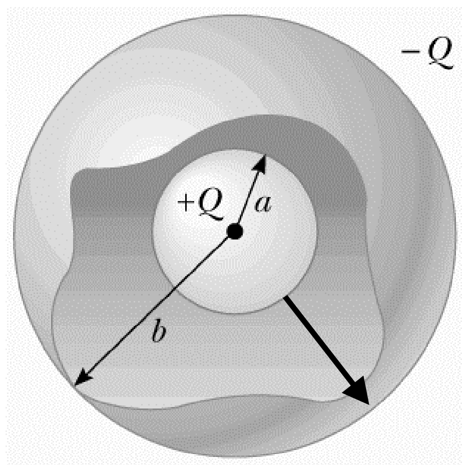

**單位面積的電容可寫為** $C'=\frac{C}{A}=\frac{e_0}{d}$

**注意$e_0$的單位：** $e_0$=8.85×10$^{-12}$ C$^2$/Nm$^2$=8.85×10$^{-12}$ F/m=8.85 pF/m

**假如**$d$=1mm，**單位面積的電容為** 8.85×10$^3$ pF/m$^2$=8.85 nF/m$^2$
1F**電容需面積**(1/8.85 × 10$^{-9}$) m$^2$~ 10$^8$ m$^2$!!

**例題** **球形電容**(a spherical capacitor)

$$e_0\oint \mathbf{E}\cdot d\mathbf{A}=Q=e_0EA=e_0E4pr^2$$
$$\mathbf{E}=\frac{Q}{4pe_0r^2}\hat{\mathbf{r}}$$
$$V=-\int_-^+\mathbf{E}\cdot d\mathbf{s}=\int_+^-\mathbf{E}\cdot d\mathbf{s}$$



## 電磁學

$$V=-\int_{-}^{+}\mathbf{E}\cdot d\mathbf{s}=\int_{+}^{-}\mathbf{E}\cdot d\mathbf{s}=\int_{a}^{b}\frac{Q}{4\pi\varepsilon_0 r^2}dr$$

$$=-\frac{Q}{4\pi\varepsilon_0}\frac{1}{r}\bigg|_a^b=\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)=\frac{Q}{4\pi\varepsilon_0}\frac{b-a}{ab}$$

$$C=\frac{Q}{V}=4\pi\varepsilon_0\frac{ab}{b-a}$$

***單一導電球的電容***

**我們可以假想另一極在無窮遠處，即**$b\to\infty$**，**$a=R$

$$C=4\pi\varepsilon_0\frac{a}{1-a/b}\to 4\pi\varepsilon_0 R$$

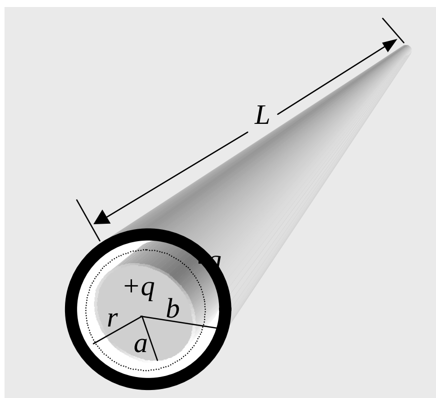


## 電磁學

**例題**　**同軸電纜**(coaxial cable)**之電容**

$$\varepsilon_0\oint\mathbf{E}\cdot d\mathbf{A}=q=\varepsilon_0 E 2\pi rL$$

$$E=\frac{q}{2\pi\varepsilon_0 Lr}$$




$$V=-\int_{-}^{+}\mathbf{E}\cdot d\mathbf{s}=\int_{+}^{-}\mathbf{E}\cdot d\mathbf{s}=\int_{a}^{b}\frac{q}{2\pi\varepsilon_0 Lr}dr=\frac{q}{2\pi\varepsilon_0 L}\ln r\bigg|_a^b$$

$$=\frac{q}{2\pi\varepsilon_0 L}\ln\left(\frac{b}{a}\right)$$

$$C=\frac{q}{V}=2\pi\varepsilon_0\frac{L}{\ln\left(\frac{b}{a}\right)}$$

**單位長度的電容**

$$C'=\frac{C}{L}=\frac{2\pi\varepsilon_0}{\ln\left(\frac{b}{a}\right)}$$

## 電磁學

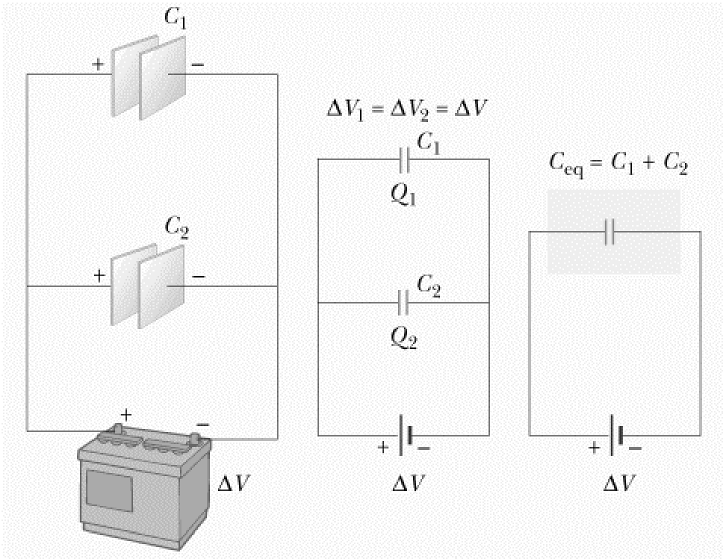
(3) **電容之並聯**(in parallel)**與串聯**(in series)

(a)**電容之並聯**(capacitors in parallel)

**並聯之電容器具有相同之電位差，即**

$$\Delta V_1 = \Delta V_2 = \Delta V_3 = \cdots = \Delta V$$
$$Q_1 = C_1 \Delta V_1, \quad Q_2 = C_2 \Delta V_2, \quad Q_3 = C_3 \Delta V_3, \cdots$$

$$Q = Q_1 + Q_2 + Q_3 + \cdots = \sum Q_i$$
$$= C_1 \Delta V_1 + C_2 \Delta V_2 + C_3 \Delta V_3 + \cdots$$
$$= \sum C_i \Delta V_i = \Delta V \sum C_i$$




$$\boxed{C_{eq} = \frac{Q}{\Delta V} = C_1 + C_2 + C_3 + \cdots = \sum C_i}$$



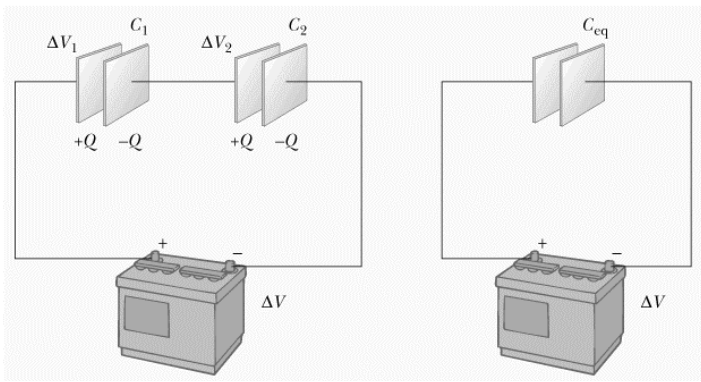
## 電磁學

(b)**電容之串聯**(capacitors in series)

**串聯之電容器之總電位差為個別電容兩端電位差之和，且每一電容器之帶電量均相同。**

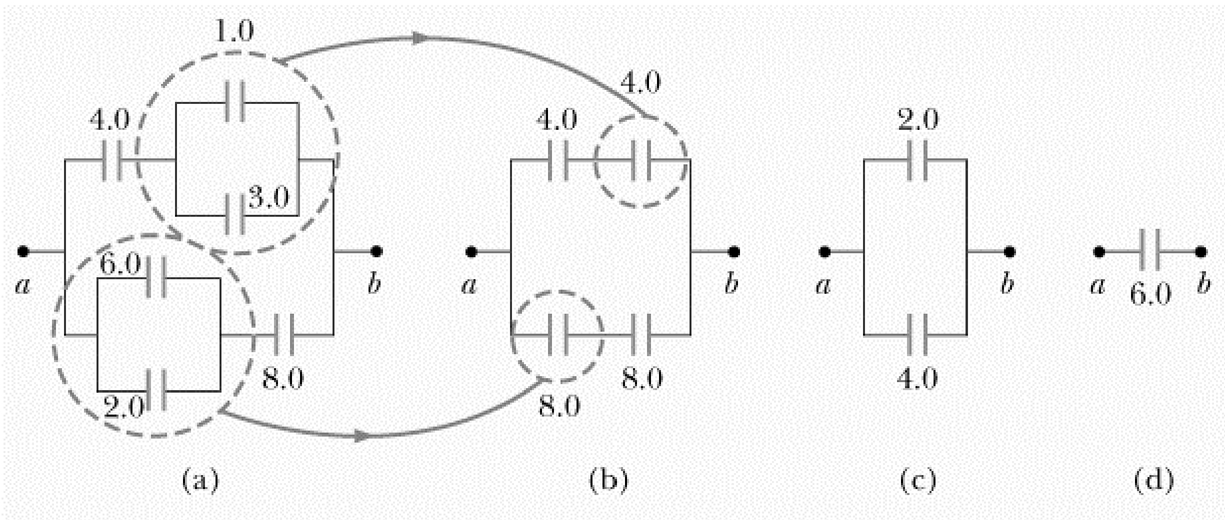
$$Q_1 = Q_2 = Q_3 = \cdots = Q$$
$$\Delta V = \Delta V_1 + \Delta V_2 + \Delta V_3 + \cdots = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} + \cdots = \frac{Q}{C_{eq}}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$$
$$= \sum \frac{1}{C_i}$$



## 電磁學

### 例題

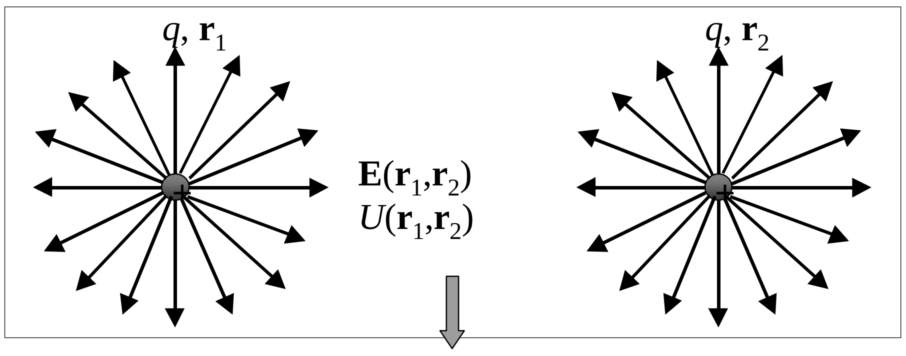


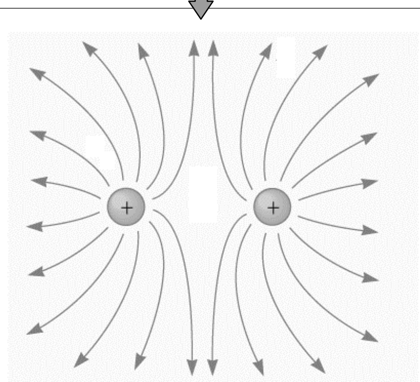



## 電磁學

### 電能之儲存

(1)電能儲存在哪裡？先從一個簡單的例子來看：



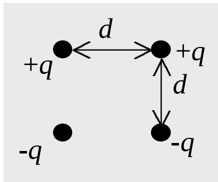



$\mathbf{E}(\mathbf{r}'_1,\mathbf{r}'_2)$
$U(\mathbf{r}'_1,\mathbf{r}'_2)$

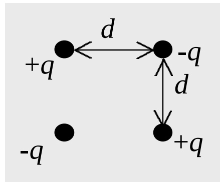
## 電磁學

**前後電場分布不同而已，帶電體本身結構不變，因此可推知位能之變化來自電場的分布不同，即*電位能是儲存在電場(所在的空間)之中*！！**

**再看一個例子**




**和**




**能量不同，**

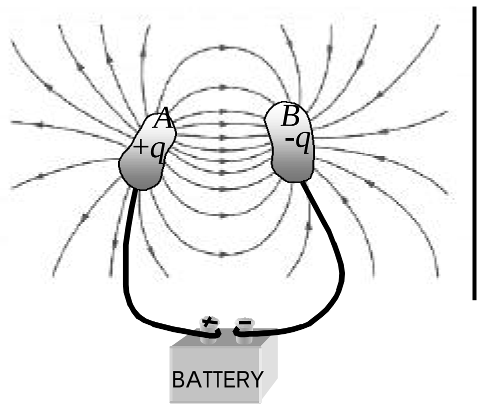
**和形成之次序無關，和排列位置有關，而排列位置影響到電場的分布。**

(2) **電容所儲存之電能**

**考慮**$dq$**由**$B$**導體移至**$A$**導體，外界做功**$Vdq$**，即**

$dU=Vdq$

$$U=\int dU=\int_0^Q Vdq=\int_0^Q \frac{q}{C}dq=\frac{1}{2}\frac{Q^2}{C}$$






## 電磁學

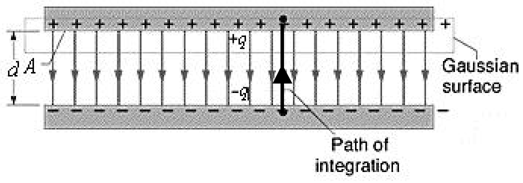
$$\boxed{U=\frac{1}{2}\frac{Q^2}{C}=\frac{1}{2}CV^2}$$

(3) **電能密度與電場**

**電能密度與電場強度有關，即空間中儲存之電能密度是當地電場強度的函數。下面我們利用簡單的平行板電容導出此關係函數。**

$$E=\frac{s}{e_0}=\frac{q}{Ae_0}, \qquad C=A\frac{e_0}{d}$$

$$U=\frac{1}{2}\frac{q^2}{C}=\frac{1}{2}\frac{d}{Ae_0}\left(AEe_0\right)^2=\frac{1}{2}e_0E^2(Ad)$$




$$u=\frac{1}{2}e_0E^2$$

**此公式適用於真空中之任意電場分布。**

$u$**為電能密度，單位為**[J/m$^3$]**，和電場強度的平方成正比。**

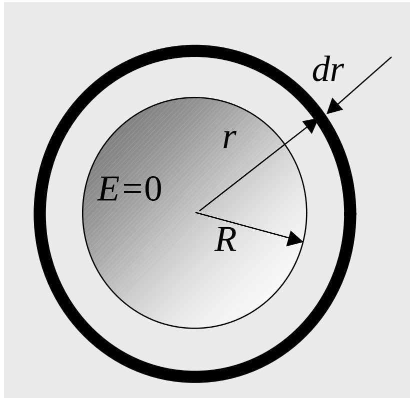
**例題** **考慮一帶電之球形導體**

$$\mathbf{E}=\left(\frac{q}{4\pi\varepsilon_0 r^2}\right)\hat{\mathbf{r}}$$

$$u=\frac{1}{2}\varepsilon_0\left(\frac{q}{4\pi\varepsilon_0 r^2}\right)^2$$

$$U=\int_{r>R} u\,du=\int_R^\infty \frac{1}{2}\varepsilon_0\left(\frac{q}{4\pi\varepsilon_0 r^2}\right)^2 4\pi r^2 dr$$

$$=\int_R^\infty \frac{1}{2}\left(\frac{q^2}{4\pi\varepsilon_0}\right)\frac{dr}{r^2}=\frac{q^2}{8\pi\varepsilon_0 R}$$




**另解**

$$C=4\pi R\varepsilon_0$$

$$U=\frac{1}{2}\frac{q^2}{C}=\frac{q^2}{8\pi\varepsilon_0 R}$$





**例題**

Two capacitors $C_1$ and $C_2$ (where $C_1 > C_2$) are charged to the same initial potential difference $\Delta V_i$, but with opposite polarity. The charged capacitors are removed from the battery, and their plates are connected as shown in Figure a. The switches $S_1$ and $S_2$ are then closed, as shown in Figure b. (a) Find the final potential difference $\Delta V_f$ between $a$ and $b$ after the switches are closed. (b) Find the total energy stored in the capacitors before and after the switches are closed and the ratio of the final energy to the initial energy.

(a)

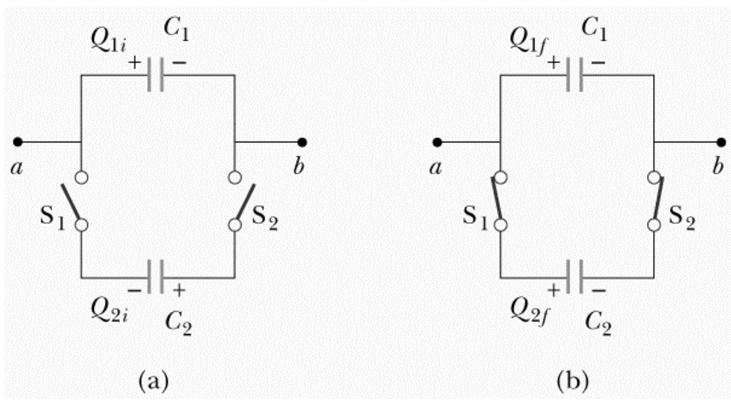
$$Q_{1i}=C_1\Delta V_i \qquad Q_{2i}=-C_2\Delta V_i$$

$$Q=Q_{1i}+Q_{2i}=(C_1-C_2)\Delta V_i$$

$$=Q_{1f}+Q_{2f}$$

$$Q_{1f}=C_1\Delta V_f \qquad Q_{2f}=C_2\Delta V_f$$

$$Q_{1f}:Q_{2f}=C_1:C_2$$



$$Q_{1f} = Q\left(\frac{C_1}{C_1 + C_2}\right) \qquad Q_{2f} = Q\left(\frac{C_2}{C_1 + C_2}\right)$$

$$\Delta V_f = \frac{Q_{1f}}{C_1} = \frac{Q_{2f}}{C_2} = \frac{Q}{C_1 + C_2} = \left(\frac{C_1 - C_2}{C_1 + C_2}\right)\Delta V_i$$

(b)

$$U_i = \tfrac{1}{2}C_1(\Delta V_i)^2 + \tfrac{1}{2}C_2(\Delta V_i)^2 = \tfrac{1}{2}(C_1 + C_2)(\Delta V_i)^2$$

$$U_f = \frac{1}{2}\frac{Q^2}{(C_1 + C_2)} = \frac{1}{2}\frac{(C_1 - C_2)^2(\Delta V_i)^2}{(C_1 + C_2)}$$

$$\frac{U_f}{U_i} = \left(\frac{C_1 - C_2}{C_1 + C_2}\right)^2$$