

4. The speed is $v = (27.0 \text{ km/h})(10^3 \text{ m/km})(1 \text{ h}/3600 \text{ s}) = 7.50 \text{ m/s}$ (direction CCW)

The circumference is $2\pi R = 2\pi(172 \text{ m}) = 1080 \text{ m}$.

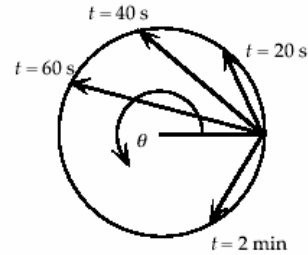
The time for a complete lap is

$$T = 2\pi R/v = 1080 \text{ m}/(7.50 \text{ m/s}) = 144 \text{ s}.$$

To find the angle from the initial position:

$$\theta = (360^\circ/T)t = (360^\circ/144 \text{ s})t.$$

Thus $\theta_{20} = 50^\circ$; $\theta_{40} = 100^\circ$; $\theta_{60} = 150^\circ$; $\theta_{120} = 300^\circ$.



9. Given $\vec{r} = (4 \text{ m}) \cos(\pi t/T) \hat{i} - (4 \text{ m}) \sin(\pi t/T) \hat{j}$;

$$\vec{r}_{T/3} = (4 \text{ m}) \cos(\pi T/3T) \hat{i} - (4 \text{ m}) \sin(\pi T/3T) \hat{j} = \boxed{(2.0 \hat{i} - 3.5 \hat{j}) \text{ m}}$$

$$d = [(2.0 \text{ m})^2 + (3.5 \text{ m})^2]^{1/2} = \boxed{4.0 \text{ m}}$$

$$\vec{r}_{T/2} = \boxed{-(4.0 \text{ m}) \hat{j}}; \quad d = \boxed{4.0 \text{ m}}$$

$$\vec{r}_{2T} = \boxed{(4.0 \text{ m}) \hat{i}}; \quad d = \boxed{4.0 \text{ m}}$$

The angle is found from $\tan \theta = y/x = -4 \sin(\pi t/T)/[4 \cos(\pi t/T)] = -\tan(\pi t/T)$, so $\theta(t) = \boxed{-\pi t/T}$.

The particle is traveling CW around a circle.

14. From the definition of velocity, we have

$$v_x = dx/dt = d[A \cos(\omega t)]/dt = \boxed{-A\omega \sin(\omega t)};$$

$$v_y = dy/dt = d[A \sin(\omega t)]/dt = \boxed{+A\omega \cos(\omega t)}.$$

From the definition of acceleration, we have

$$a_x = dv_x/dt = d[-A\omega \sin(\omega t)]/dt = \boxed{-A\omega^2 \cos(\omega t)};$$

$$a_y = dv_y/dt = d[+A\omega \cos(\omega t)]/dt = \boxed{-A\omega^2 \sin(\omega t)}.$$

37. (a) $R = v_0^2 \sin(2\theta_0)/g$; $h = \frac{1}{2}(v_0^2 \sin^2 \theta_0)/g$. Thus

$$R/h = 2 \sin(2\theta_0)/\sin^2 \theta_0 = 2(2 \sin \theta_0 \cos \theta_0)/\sin^2 \theta_0 = 4 \cot \theta_0.$$

- (b) The angle at which the range is maximum is found from $dR/d\theta_0 = 0$:

$$dR/d\theta_0 = (v_0^2/g) \cos(2\theta_0) (2) = 0; \quad \cos(2\theta_0) = 0; \quad 2\theta_0 = \frac{1}{2}\pi; \quad \theta_0 = \frac{1}{4}\pi = 45^\circ.$$

$$\text{Thus } R = 4h \cot 45^\circ = 4h; \quad h = \frac{1}{4}R.$$

44. (a) At the highest point, the vertical velocity $v_y = 0$.

The speed will be $v_x = v_{0x} = (28 \text{ m/s}) \cos 50^\circ = \boxed{18 \text{ m/s}}$.

- (b) At the highest point: $v_y = v_{0y} + a_y t_1$; $0 = (28 \text{ m/s}) \sin 50^\circ + (-9.8 \text{ m/s}^2)t_1$, which gives $t_1 = 2.2 \text{ s}$.

The height is $h = y_0 + v_{0y} t_1 + \frac{1}{2} a_y t_1^2 = 0 + (28 \text{ m/s}) \sin 50^\circ (2.2 \text{ s}) + \frac{1}{2} (-9.8 \text{ m/s}^2)(2.2 \text{ s})^2 = \boxed{24 \text{ m}}$.

- (c) Because the time to fall 24 m is the same as the time to rise 24 m: $t_2 = t_1 = 2.2 \text{ s}$. The speed will be

$$|v_y| = |v_{0y} + a_y t_2| = |0 + (-9.8 \text{ m/s}^2)(2.2 \text{ s})| = \boxed{21 \text{ m/s}}, \quad \text{which is } (28 \text{ m/s}) \sin 50^\circ.$$

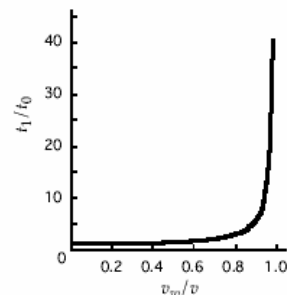
64. (a) With no wind, the speed in each direction will be v , so the time is $t_0 = 2L/v$.
 (b) With a north wind, her speed while cycling north will be $v - v_w$ and returning will be $v + v_w$.
 Thus her total time will be

$$t_1 = \frac{L}{v - v_w} + \frac{L}{v + v_w} = \frac{L(v + v_w + v - v_w)}{(v^2 - v_w^2)} = \frac{2Lv}{(v^2 - v_w^2)} = \frac{t_0}{1 - (v_w/v)^2}$$

- (c) If $v_w \ll v$, then

$$t_1 = \frac{2Lv}{v^2[1 - (v_w/v)^2]} = \frac{2L}{v} [1 + (v_w/v)^2] = t_0 [1 + (v_w/v)^2]$$

- (d) At $v_w = v$, the time becomes infinite because the cyclist will have zero velocity with respect to the ground on the first part of her trip and will never move.



69. We can express the data as $r_e = 6.4 \times 10^3$ km and $r_o = 1.5 \times 10^8$ km. Because we know the periods of the two motions, we can find the corresponding speeds from $v = r\omega = r(2\pi/T)$:

$$v_{\text{rot}} = r_e(2\pi/T_e) = (6.4 \times 10^3 \text{ km})[2\pi/(24 \text{ h})(3600 \text{ s/h})] = 0.465 \text{ km/s},$$

$$v_{\text{orb}} = r_o(2\pi/T_o) = (1.5 \times 10^8 \text{ km})[2\pi/(365 \text{ d})(24 \text{ h})(3600 \text{ s/h})] = 30.0 \text{ km/s}.$$

- (a) At the nearest point, the velocities are in opposite directions, so

$$v_1 = v_{\text{orb}} - v_{\text{rot}} = 30.0 \text{ km/s} - 0.465 \text{ km/s} = \boxed{29.5 \text{ km/s}}$$

- (b) At the farthest point, the velocities are in the same direction, so

$$v_2 = v_{\text{orb}} + v_{\text{rot}} = 30.0 \text{ km/s} + 0.465 \text{ km/s} = \boxed{30.5 \text{ km/s}}$$

- (c) At the midway point, the velocities are perpendicular, so

$$v_3 = (v_{\text{orb}}^2 + v_{\text{rot}}^2)^{1/2} = [(30.0 \text{ km/s})^2 + (0.465 \text{ km/s})^2]^{1/2} = \boxed{30.0 \text{ km/s}}$$

71. We choose a coordinate system with the origin at the tee, x horizontal and y up.

The horizontal motion is $x = v_{0x}t_1 = 155 \text{ m} = v_0 \cos 65^\circ t_1$, or $v_0 t_1 = 367 \text{ m}$.

The vertical motion is $y = y_0 + v_{0y}t_1 + \frac{1}{2}a_y t_1^2$; $4.0 \text{ m} = 0 + v_0 \sin 65^\circ t_1 + \frac{1}{2}(-9.8 \text{ m/s}^2)t_1^2$.

- (a) When these two equations are solved simultaneously, we get $t_1 = 8.2 \text{ s}$ and $v_0 = \boxed{45 \text{ m/s at } 65^\circ}$.

- (b) Because $v_y = 0$ at the maximum height, we can find the time to reach this height from

$$v_y = v_{0y} + a_y t_2; 0 = (44.8 \text{ m/s}) \sin 65^\circ + (-9.8 \text{ m/s}^2)t_2, \text{ which gives } t_2 = 4.14 \text{ s}.$$

(Note that this is less than $\frac{1}{2}t_1$ because of the slope of the ground.) The height above the green is

$$y_{\text{max}} = 0 + (44.8 \text{ m/s})(\sin 65^\circ)(4.14 \text{ s}) - \frac{1}{2}(9.8 \text{ m/s}^2)(4.14 \text{ s})^2 = \boxed{84 \text{ m}}$$

75. We choose a coordinate system with the origin at the base of the mast, x horizontal and y up. At release the hammer is at $x = 0, y = 26 \text{ m}$ and will have the horizontal velocity of the top of the mast.

The time of fall is found from

$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2;$$

$$0 = 26 \text{ m} + 0 + \frac{1}{2}(-9.8 \text{ m/s}^2)t^2, \text{ which gives } t = 2.3 \text{ s}.$$

The horizontal motion is

$$x = v_{0x}t = (3.6 \text{ m/s})(2.3 \text{ s}) = 8.3 \text{ m}.$$

Because this is less than half the width of the ship, the hammer **will hit the deck** which is assumed to be horizontal at impact.

78. Using the coordinate system shown in the diagram, we can write the equations of motion as

$$\begin{aligned}x &= v_0 \cos \theta t \quad \text{and} \\y &= y_0 + v_0 \sin \theta t + \frac{1}{2} a_y t^2, \quad \text{or} \\0 &= h_0 + v_0 \sin \theta t + \frac{1}{2} (-g) t^2.\end{aligned}$$

The y -equation yields a quadratic equation for the time to reach the ground, t :

$$t^2 - [(2v_0 \sin \theta)/g] t - 2h_0/g = 0.$$

The solutions are

$$t = + (2v_0 \sin \theta / 2g) \pm \frac{1}{2} [(2v_0 \sin \theta / g)^2 + (8h_0/g)]^{1/2}.$$

Because the cannonball starts at $t = 0$, the physical answer is the positive result:

$$t = + (v_0/g) [\sin \theta + (\sin^2 \theta + 2h_0g/v_0^2)^{1/2}].$$

The horizontal range R is

$$R = v_0 (\cos \theta) \left(\frac{v_0}{g} \right) \left[\sin \theta + \left(\sin^2 \theta + \frac{2h_0g}{v_0^2} \right)^{1/2} \right].$$

To find the angle θ at which R is a maximum, we take the derivative $dR/d\theta$ and set it equal to zero:

$$\frac{dR}{d\theta} = - \left(\frac{v_0^2}{g} \right) (\sin \theta) \left[\sin \theta + \left(\sin^2 \theta + \frac{2h_0g}{v_0^2} \right)^{1/2} \right] + \left(\frac{v_0^2}{g} \right) (\cos \theta) \left[\cos \theta + \frac{1}{2} \left(\sin^2 \theta + \frac{2h_0g}{v_0^2} \right)^{-1/2} (2 \sin \theta \cos \theta) \right] = 0.$$

This reduces to

$$-\sin^2 \theta + \cos^2 \theta + \frac{-\sin^3 \theta - \left(\frac{2h_0g}{v_0^2} \right) \sin \theta + \sin \theta \cos^2 \theta}{\sqrt{\sin^2 \theta + 2h_0g/v_0^2}} = 0.$$

If we let $2h_0g/v_0^2 = D$, use $\cos^2 \theta = 1 - \sin^2 \theta$ and let $z = \sin \theta$, we can get an equation in z :

$$1 - 2z^2 + (-z^3 - Dz + z - z^3)/(z^2 + D)^{1/2} = 0.$$

After some algebraic manipulation, including squaring, this reduces to

$$z^2 = 1/(D+2), \quad \text{or} \quad \sin^2 \theta = \frac{1}{2} v_0^2 / (gh_0 + v_0^2).$$

