

8. For the stationary brick, with no acceleration, we can write

$$\Sigma F_x = ma_x:$$

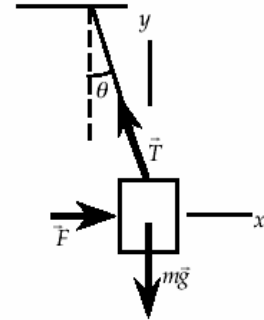
$$P - T \sin \theta = 0; \quad 12 \text{ N} - T \sin 25^\circ = 0, \text{ which gives}$$

$$T = 28 \text{ N}.$$

$$\Sigma F_y = ma_y:$$

$$T \cos \theta - mg = 0; \quad (28 \text{ N}) \cos 25^\circ - m(9.8 \text{ m/s}^2) = 0, \text{ which gives}$$

$$m = \boxed{2.6 \text{ kg}}$$



12. Forces are drawn for each of the blocks for the situation when m_1 leaves the floor (no normal force). Because the string doesn't stretch, the tension is the same at each end of the string, and the accelerations of the blocks have the same magnitude. Note that we take the positive direction in the direction of the acceleration for each block.

- (a) We write $\Sigma \vec{F} = m\vec{a}$ from the force diagram for each block:

$$y\text{-component (block 1): } T - m_1g = m_1a.$$

$$y\text{-component (block 2): } m_2g - T = m_2a.$$

By adding the equations, we find the acceleration:

$$\begin{aligned} a &= (m_2 - m_1)g / (m_1 + m_2) \\ &= (1.700 \text{ kg} - 1.650 \text{ kg})(9.8 \text{ m/s}^2) / (1.70 \text{ kg} + 1.65 \text{ kg}) \\ &= \boxed{0.15 \text{ m/s}^2} \text{ for both blocks.} \end{aligned}$$

- (b) For the motion of block 2:

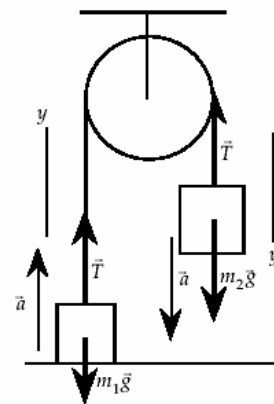
$$v_2^2 = v_{02}^2 + 2a(y_2 - y_{02}) = 0 + 2(0.15 \text{ m/s}^2)(2.15 \text{ m} - 0),$$

$$\text{which gives } v_2 = \boxed{0.80 \text{ m/s}}$$

(Note: block 1 has the same speed. Once block 2 hits the floor, $T \rightarrow 0$ and the motions of the two blocks will differ.)

- (c) To find the time to reach the floor, for block 2:

$$v_2 = v_{02} + at; \quad 0.803 \text{ m/s} = 0 + (0.15 \text{ m/s}^2)t, \text{ which gives } t = \boxed{5.4 \text{ s}}$$



17. The force diagrams for each of the masses and the movable pulley are shown. Note that we take down as positive and the indicated accelerations are relative to the fixed pulley. A downward acceleration of m_1 means an upward acceleration of the movable pulley. If we call a_r the acceleration of m_2 with respect to the movable pulley, then

$$a_2 = a_r - a_1 \quad \text{and} \quad a_3 = -a_r - a_1,$$

because the acceleration of m_3 with respect to the pulley must be the negative of m_2 's acceleration with respect to the pulley.

If the mass of the pulley is negligible, we write $\Sigma F_y = ma_y$:

$$2T_2 - T_1 = (0)(-a_1), \text{ so } 2T_2 = T_1.$$

For each of the masses, for $\Sigma F_y = ma_y$ we get

$$\text{mass } m_1: \quad m_1 g - T_1 = m_1 a_1,$$

$$\text{mass } m_2: \quad m_2 g - T_2 = m_2 a_2 = m_2 (a_r - a_1),$$

$$\text{mass } m_3: \quad m_3 g - T_2 = m_3 a_3 = m_3 (-a_r - a_1).$$

We have four equations for the four unknowns:

$$T_1, T_2, a_r, \text{ and } a_1.$$

After some careful algebra, we get

$$a_1 = [(m_1 m_2 + m_1 m_3 - 4m_2 m_3) / (m_1 m_2 + m_1 m_3 + 4m_2 m_3)]g;$$

$$a_r = [2m_1(m_2 - m_3) / (m_1 m_2 + m_1 m_3 + 4m_2 m_3)]g;$$

$$T_1 = [8m_1 m_2 m_3 / (m_1 m_2 + m_1 m_3 + 4m_2 m_3)]g; \text{ and}$$

$$T_2 = [4m_1 m_2 m_3 / (m_1 m_2 + m_1 m_3 + 4m_2 m_3)]g.$$

We can now find the other accelerations:

$$a_2 = [(m_1 m_2 - 3m_1 m_3 + 4m_2 m_3) / (m_1 m_2 + m_1 m_3 + 4m_2 m_3)]g;$$

$$a_3 = [(-3m_1 m_2 + m_1 m_3 + 4m_2 m_3) / (m_1 m_2 + m_1 m_3 + 4m_2 m_3)]g.$$

If $m_2 = m_3 \neq m_1$:

$$a_r = 0;$$

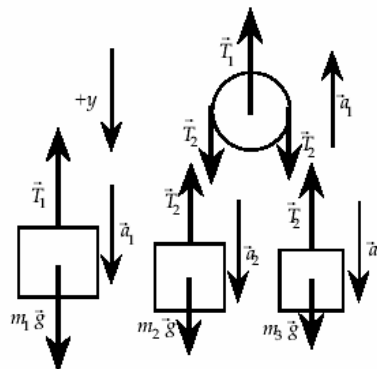
$$a_1 = [(m_1 - 2m_2) / (m_1 + 2m_2)]g$$

$$T_1 = [4m_1 m_2 / (m_1 + 2m_2)]g;$$

$$T_2 = [2m_1 m_2 / (m_1 + 2m_2)]g;$$

$$a_2 = a_3 = -a_1 = [(2m_2 - m_1) / (m_1 + 2m_2)]g.$$

Thus m_2 and m_3 have the same acceleration as the pulley. Note that neither tension is equal to an mg !



24. (a) With the block as the system, there is no acceleration.

Using the force diagram, we can write $\Sigma \vec{F} = m\vec{a}$:

$$\text{x-component: } T_1 - mg \sin \theta - f_{k1} = 0;$$

$$\text{y-component: } F_{N1} - mg \cos \theta = 0.$$

$$\text{Thus } T_1 = mg \sin \theta + \mu_k mg \cos \theta$$

$$= mg(\sin \theta + \mu_k \cos \theta)$$

$$= (25 \text{ kg})(9.8 \text{ m/s}^2)(\sin 25^\circ + 0.4 \cos 25^\circ) = \boxed{1.9 \times 10^2 \text{ N}}.$$

- (b) We have a new force diagram, so $\Sigma \vec{F} = m\vec{a}$ becomes

$$\text{x-component: } T_2 \cos(\theta - \phi) - mg \sin \theta - \mu_k F_{N2} = 0;$$

$$\text{y-component: } T_2 \sin(\theta - \phi) + F_{N2} - mg \cos \theta = 0.$$

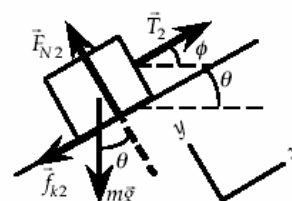
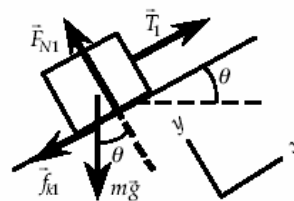
If we substitute the data, we get

$$T_2 \cos(40^\circ - 25^\circ) - (25 \text{ kg})(9.8 \text{ m/s}^2) \sin 25^\circ - (0.4)F_{N2} = 0;$$

$$T_2 \sin(40^\circ - 25^\circ) + F_{N2} - (25 \text{ kg})(9.8 \text{ m/s}^2) \cos 25^\circ = 0.$$

When these two equations are solved for the two unknowns, we get

$$F_{N2} = 1.8 \times 10^2 \text{ N} \quad \text{and} \quad T_2 = \boxed{1.8 \times 10^2 \text{ N}}.$$



30. Until the box moves, friction is static, opposing the impending motion.

$$(96 \text{ km/h}) / (3600 \text{ s/h}) = 0 + (6.9 \text{ m/s}^2)t, \text{ which gives } t = \boxed{3.9 \text{ s}}$$

We write $\sum \vec{F} = m\vec{a}$ from the force diagram for the mass:

$$x\text{-component: } T \cos \theta - f_s = 0;$$

$$y\text{-component: } T \sin \theta + F_N - mg = 0.$$

When the box is on the verge of moving, the friction force is maximum, so we have

$$f_s = f_{s,\text{max}} = \mu_s F_N.$$

When this is put into the x -component equation, we can solve for T to get

$$T = \mu_s mg / (\cos \theta + \mu_s \sin \theta) \\ = 0.40(2.0 \text{ kg})(9.8 \text{ m/s}^2) / (\cos 50^\circ + 0.40 \sin 50^\circ) = \boxed{8.3 \text{ N}}$$

36. (a) We write $\sum \vec{F} = m\vec{a}$ from the force diagram for the top mass:

$$x\text{-component: } 0 = m_1 a_1;$$

$$y\text{-component: } F_{N1} - m_1 g = 0.$$

We write $\sum \vec{F} = m\vec{a}$ from the diagram for the bottom mass:

$$x\text{-component: } F = m_2 a_2;$$

$$y\text{-component: } F_{N2} - F_{N1} - m_2 g = 0.$$

Thus we have $F_{N1} = m_1 g$; $F_{N2} = F_{N1} + m_2 g$; and

$$\boxed{a_1 = 0; \quad a_2 = F/m_2}$$

- (b) We write $\sum \vec{F} = m\vec{a}$ for the combined masses:

$$x\text{-component: } F = (m_1 + m_2)a;$$

Thus $a = \boxed{a_1 = a_2 = F/(m_1 + m_2)}$.

- (c) Assuming the top mass is sliding to the left, we have

$$\vec{F}_{N1} = m_1 g \hat{j} \text{ (up) and } \vec{f}_1 = \mu_k m_1 g \hat{i} \text{ (right), so}$$

$$\boxed{F \text{ (lower on upper)} = (\mu_k m_1 g) \hat{i} + (m_1 g) \hat{j}}$$

- (d) The y -component equations are the same as in part (a).

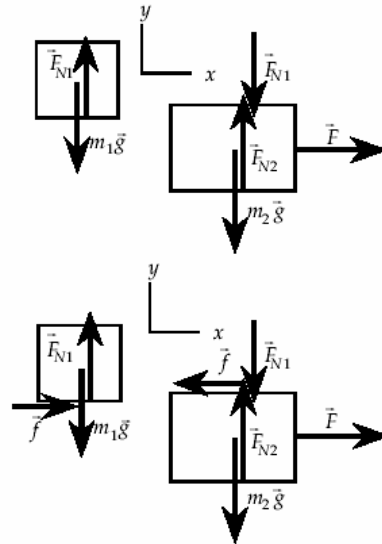
We write $\sum \vec{F} = m\vec{a}$ from the force diagram for the top mass:

$$x\text{-component: } \mu_k m_1 g = m_1 a_1.$$

We write $\sum \vec{F} = m\vec{a}$ from the diagram for the bottom mass:

$$x\text{-component: } F - \mu_k m_1 g = m_2 a_2.$$

Thus $\boxed{a_1 = \mu_k g}$ and $\boxed{a_2 = (F - \mu_k m_1 g)/m_2}$.



46. From $F_d = -Av$, we find the dimensions of A :

$$[A] = [F]/[v] = [MLT^{-2}]/[LT^{-1}] = [MT^{-1}].$$

We assume that the time will be a function of A , g and m , so we write $t = m^\alpha g^\beta A^\gamma$.

In terms of dimensions, this gives

$$[t] = [m]^\alpha [g]^\beta [A]^\gamma, \quad \text{or} \quad [T] = [M]^\alpha [LT^{-2}]^\beta [MT^{-1}]^\gamma.$$

Equating the exponents of the various dimensions, we get $1 = -2\beta - \gamma$, $0 = \alpha + \gamma$, and $0 = \beta$.

Thus $\beta = 0$, $\gamma = -1$, and $\alpha = +1$, so $\boxed{t \approx m/A}$.

51. If the automobile does not skid, the friction is static, with $f_s \leq \mu_s F_N$. At high speed f_s will be down the incline. Note that we take a coordinate system with the x -axis in the direction of the centripetal acceleration. We write $\sum \vec{F} = m\vec{a}$ from the force diagram for the auto:

$$x\text{-component: } F_N \sin \theta + f_s \cos \theta = ma = mv^2/R;$$

$$y\text{-component: } F_N \cos \theta - f_s \sin \theta - mg = 0.$$

The speed is maximum when $f_s = f_{s,\max} = \mu_s N$.

From the y -equation we get

$$F_N \cos \theta - \mu_s F_N \sin \theta = mg, \text{ or } F_N = mg/(\cos \theta - \mu_s \sin \theta).$$

From the x -equation we get

$$v_{\max}^2/R = g(\sin \theta + \mu_s \cos \theta)/(\cos \theta - \mu_s \sin \theta).$$

$$v_{\max}^2 = (150 \text{ m})(9.8 \text{ m/s}^2)(\sin 18^\circ + 0.3 \cos 18^\circ)/(\cos 18^\circ - 0.3 \sin 18^\circ), \text{ which gives } v_{\max} = \boxed{32 \text{ m/s}}$$

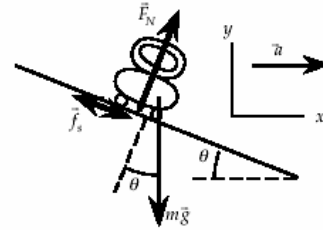
At low speed, the automobile will tend to slide down the incline, so f_s will be up the incline.

The speed is minimum when $f_s = f_{s,\max} = \mu_s F_N$.

If we change the sign of f_s in the equations, we get

$$F_N = mg/(\cos \theta + \mu_s \sin \theta) \text{ and } v_{\min}^2/R = mg(\sin \theta - \mu_s \cos \theta)/(\cos \theta + \mu_s \sin \theta). \text{ Thus}$$

$$v_{\min}^2 = (150 \text{ m})(9.8 \text{ m/s}^2)(\sin 18^\circ - 0.3 \cos 18^\circ)/(\cos 18^\circ + 0.3 \sin 18^\circ), \text{ which gives } v_{\min} = \boxed{5.8 \text{ m/s}}$$



55. While the stone does not slide on the turntable, the static friction force provides the centripetal acceleration:

$$f_s = ma_r = mR\omega^2.$$

Thus a larger friction force is needed at larger R . At the critical distance the friction force is maximum

$$f_s = f_{s,\max} = \mu_s mg, \text{ so we have}$$

$$\mu_s mg = mR\omega^2, \text{ or}$$

$$\mu_s = R\omega^2/g = (0.21 \text{ m})[(33 \text{ rev/min})(2\pi \text{ rad/rev})/(60 \text{ s/min})]^2/(9.8 \text{ m/s}^2) = \boxed{0.26}$$

63. The bob is subject to three forces: T and mg . The net force is horizontal and points towards the center of its circular path. We write $\sum = m\vec{a}$ for the bob:

$$x\text{-component: } T \sin \theta = ma_x = mR\omega^2 = m(L \sin \theta) \omega^2;$$

$$y\text{-component: } T \cos \theta - mg = ma_y = 0.$$

Solve for L , the length of the string:

$$L = g/\omega^2 \cos \theta = (9.8 \text{ m/s}^2)/[(\frac{1}{2}\pi \text{ rad/s})^2 \cos 30^\circ] = \boxed{4.5 \text{ m}}$$

65. We write $\sum \vec{F} = m\vec{a}$ from the force diagram for the block:

$$x\text{-component: } F_N \cos \theta = mr\omega^2;$$

$$y\text{-component: } F_N \sin \theta - mg = 0.$$

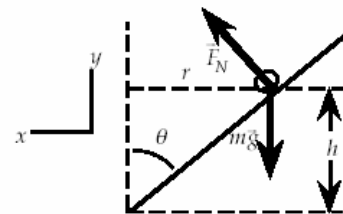
Combining these we get

$$r = g/(\omega^2 \tan \theta). \text{ Then}$$

$$h = r/\tan \theta = g/(\omega^2 \tan^2 \theta)$$

$$= (9.8 \text{ m/s}^2)/[(3.8 \text{ rad/s})^2 \tan^2 44^\circ]$$

$$= \boxed{0.72 \text{ m}}$$



66. (a) We write $\sum \vec{F} = m\vec{a}$ from the force diagram for the mass:

$$x\text{-component: } T_1 \cos 30^\circ - T_2 \cos 30^\circ = mr\omega^2;$$

$$y\text{-component: } T_1 \sin 30^\circ - T_2 \sin 30^\circ - mg = 0.$$

From the y -equation we get:

$$(150 \text{ N}) \sin 30^\circ - T_2 \sin 30^\circ - (5.0 \text{ kg})(9.8 \text{ m/s}^2) = 0,$$

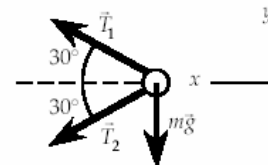
$$\text{which gives } T_2 = \boxed{52 \text{ N}}.$$

- (b) From the x -equation we get

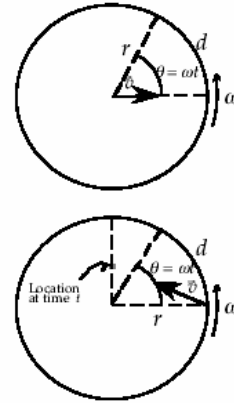
$$(150 \text{ N}) \cos 30^\circ + (52 \text{ N}) \cos 30^\circ = (5.0 \text{ kg})(1.0 \text{ m}) \cos 30^\circ \omega^2;$$

$$\text{which gives } \omega = 6.36 \text{ rad/s}.$$

$$\text{The time for one circuit is then } t = 2\pi/\omega = \boxed{0.99 \text{ s}}.$$



72. (a) In the time $t = r/v$ that it takes the ball to reach the rim of the platform, it will have rotated an angle $\theta = \omega t$. A point on the rim will have moved a distance $d = r\theta = r\omega t = v\omega t^2$.
- (b) To the observer on the platform, the ball will have moved opposite to the direction of rotation a distance d in a time t . The apparent acceleration can be found from $d = v_{0\perp}t + \frac{1}{2}at^2$; $a = 2d/t^2 = 2(v\omega t^2)/t^2 = 2v\omega$, perpendicular to v .
- (c) In the inertial frame, the ball thrown from the rim also has the initial tangential velocity of the rim, $v = r\omega$. In this frame, as the ball moves in a straight line, it will not go to the center of the platform but will go to the right of the direction of throw. To an observer on the platform, the ball will have an acceleration to the right of the radial line perpendicular to the motion.



77. (a) Because the length of the rope is constant, when m_1 moves up Δx_1 , the segment above m_1 decreases by Δx_1 , and each segment above m_2 must increase by one-half that amount:

$$\Delta x_2 = -\frac{1}{2}\Delta x_1 \quad (- \text{ indicates the opposite direction}).$$

- (b) If we differentiate with respect to time twice:

$$v_2 = -\frac{1}{2}v_1;$$

$$a_2 = -\frac{1}{2}a_1.$$

We write $\sum \vec{F} = m\vec{a}$ from the force diagram for m_1 :

$$x\text{-component: } T - m_1g = m_1a_1.$$

We write $\sum \vec{F} = m\vec{a}$ from the force diagram for m_2 :

$$x\text{-component: } 2T - m_2g = m_2a_2 = -\frac{1}{2}m_2a_1.$$

By eliminating T between these equations, we get

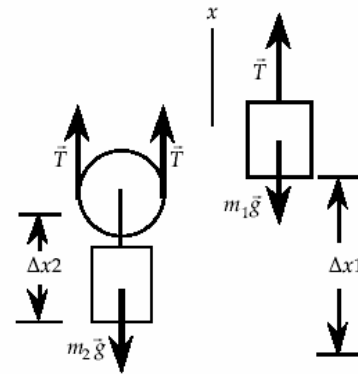
$$a_1 = -2g[(2m_1 - m_2)/(4m_1 + m_2)] \\ = -2(9.8 \text{ m/s}^2)[2(1.2 \text{ kg}) - 1.8 \text{ kg}]/[4(1.2 \text{ kg}) + 1.8 \text{ kg}]$$

$$= \boxed{-1.78 \text{ m/s}^2 \text{ (down)}}.$$

$$a_2 = -\frac{1}{2}a_1 = \boxed{+0.89 \text{ m/s}^2 \text{ (up)}}.$$

- (c) For the tension we have

$$T = m_1(g + a_1) = (1.2 \text{ kg})(9.8 \text{ m/s}^2 - 1.78 \text{ m/s}^2) = \boxed{9.6 \text{ N}}.$$



80. (a) **No**. The most likely point where the string will break is at the bottom of the circular track, not at the top. The least likely point for the string to break is at the top of the track, where the tension in the string is the lowest in magnitude.
- (b) At the bottom of the track $T - mg = ma = mv^2/R$. Set $T = 40 \text{ N}$ to obtain $v = (TR/m - gR)^{1/2} = [(40 \text{ N})(1.0 \text{ m})/(0.150 \text{ kg}) - (9.8 \text{ m/s}^2)(1.0 \text{ m})]^{1/2} = \boxed{16 \text{ m/s}}$.

83. From Problem 61 we have

$$\tan \theta = v^2 / rg = r\omega^2 / g = (\ell \sin \theta) \omega^2 / g, \text{ which gives } \omega = \boxed{[g / (\ell \cos \theta)]^{1/2}}.$$

85. The static friction force provides the centripetal acceleration.

We write $\sum \vec{F} = m\vec{a}$ from the force diagram for the bicycle:

$$x\text{-component: } f_s = mv^2/R;$$

$$y\text{-component: } F_N - mg = 0.$$

The shortest turn (smallest R) requires $f_{s,\max} = \mu_s F_N$.

Thus $R_{\min} = mv^2 / f_{s,\max} = mv^2 / \mu_s mg$

$$= v^2 / \mu_s g = (10 \text{ m/s})^2 / 0.4(9.8 \text{ m/s}^2) = \boxed{26 \text{ m}}.$$

