8. (a) $W_g = -mg\,\Delta y = -(37\text{ kg})(9.8\text{ m/s}^2)(7.5\text{ m}) = \boxed{-2.7\times10^3\text{ J}}$.

(b) The tension has the same magnitude as the gravity force:

$W_T = (37\text{ kg})(9.8\text{ m/s}^2)(7.5\text{ m}) = \boxed{+2.7\times10^3\text{ J}}$. (Net work is zero.)

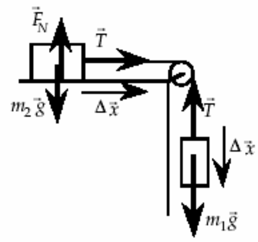
(c) The person does no work directly on the load.
The work is done on the rope as it passes through the hands.

14. (a) When we take the two blocks as the system, the tension becomes an internal force. The only force that does work is the work done by gravity on block m_1. For the work-energy theorem, we have

$$W_{net} = \Delta K$$
$$= \tfrac{1}{2}m_1v^2 - \tfrac{1}{2}m_1v_0^2 + \tfrac{1}{2}m_2v^2 - \tfrac{1}{2}m_2v_0^2$$
$$= \tfrac{1}{2}m_1v^2 + \tfrac{1}{2}m_2v^2 - 0;$$

$m_1g\,\Delta x = \tfrac{1}{2}m_1v^2 + \tfrac{1}{2}m_2v^2 - 0$, which gives

$v = \boxed{[2m_1g\,\Delta x/(m_1+m_2)]^{1/2}}$.



(b) Because all of the forces are constant, the acceleration will be constant. We find it from

$v^2 = v_0^2 + 2a\,\Delta x;$

$2m_1g\,\Delta x/(m_1+m_2) = 0 + 2a\,\Delta x$, which gives

$a = \boxed{m_1g/(m_1+m_2)}$.

20. The work done by the force of gravity on m_1 as it is lowered through a distance Δy_1 $(= l_1\theta)$ is given by $W_{g1} = m_1\,g\,\Delta\,y_1 = m_1\,g\,l_1\,\theta$, while that done on m_2 as it is lifted through a distance Δy_2 $(= l_2\theta)$ is $W_{g2} = -m_2\,g\,\Delta\,y_2 = -m_2\,g\,l_2\,\theta$. The kinetic energy of the rod must then change by

$$\Delta K = W_{g1} + W_{g2} = m_1\,g\,l_1\theta - m_2\,g\,l_2\theta = (m_1\,l_1 - m_2\,l_2)\,g\theta\,.$$

But since $m_1\,l_1 = m_2\,l_2$ we have $\Delta K = 0$, so $K_f = K_i + \Delta K = 0$, meaning that the rod is stationary after the slight rotation.

21. As the mass swings upward it slows down, meaning that it is losing kinetic energy. From the work-energy theorem we know that negative work must be done on it. In fact, two forces are exerted on the mass: the tension in the string and the weight of the mass. Since the tension always points along the string while the mass moves perpendicularly to the string the tension does not do any work on the mass. The (negative) work done by the weight of the mass is

$$W_g = -m\,g\,\Delta\,y = \boxed{-mgR\,(1-\cos\theta)}.$$

30. We assume there is no change in elevation. We find the net work from the change in kinetic energy:

$$W_{net} = \Delta K = \tfrac{1}{2}m(v_f^2 - v_i^2)$$
$$= \tfrac{1}{2}(1300\text{kg})[(0.21\text{ m/s})^2 - (0.15\text{ m/s})^2]$$
$$= \boxed{14\text{ J}}.$$

35. We find the magnitude of the projection of $\vec{A} = 3\hat{i} - 2\hat{j}$ onto the unit vector $\vec{e} = -0.6\hat{i} + 0.8\hat{j}$ from

$$|\,\vec{A}\cdot\vec{e}\,| = |(3)(-0.6) + (-2)(0.8)| = \boxed{3.4}.$$

42. The work of a variable force is found by integrating. Because the force changes at $x = 0$, the work will be the sum of two integrals:

$$W = \int_{-1.50\text{m}}^{0}(-3.00\text{ N/m})x\,dx \;+\; \int_{0}^{+1.50\text{m}}(+7.00\text{ N/m})x\,dx$$
$$= (-3.00\text{ N/m})\frac{1}{2}\left[x^2\right]_{-1.50\text{m}}^{0} + (+7.00\text{ N/m})\frac{1}{2}\left[x^2\right]_{0}^{+1.50\text{m}}$$
$$= 11.3\text{ J}\,.$$

47. The force required to stretch the spring must be opposite to the spring force and thus is
$F = +k_1 x + k_2 x^3$.
The work done by this force is

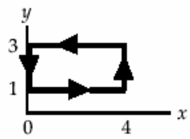
$$W = \int_{0.10\text{ m}}^{0.20\text{ m}} F\,\mathrm{d}x = \int_{0.10\text{m}}^{0.20\text{m}} (k_1 x + k_2 x^3)\,\mathrm{d}x = k_1 \frac{1}{2} x^2 \Big|_{0.10\text{ m}}^{0.20\text{ m}} + k_2 \frac{1}{4} x^4 \Big|_{0.10\text{ m}}^{0.20\text{ m}}$$

$$= (5.0\,\text{N/m})\tfrac{1}{2}\left[(0.20\text{ m})^2 - (0.10\text{ m})^2\right] + (15\,\text{N/m}^3)\tfrac{1}{4}\left[(0.20\text{ m})^4 - (0.10\text{ m})^4\right] = \quad 8.1 \times 10^{-2}\text{ J}.$$

54. In general the work of a force is $W_F = \int \vec{F} \cdot \mathrm{d}\vec{r}$. If we write $\mathrm{d}\vec{r} = \mathrm{d}x\,\hat{i} + \mathrm{d}y\,\hat{j}$, this becomes
$\int (F_x\ \mathrm{d}x + F_y\ \mathrm{d}y)$. For path 1, $\mathrm{d}y = 0$,

$$W_1 = \int_{(0,1)}^{(4,1)} \left(axy - by^2\right) \mathrm{d}x = \left(\tfrac{1}{2}ax^2y - by^2x\right)\Big|_{(0,1)}^{(4,1)}$$

$$= \tfrac{1}{2}\left(2\text{ N/m}^2\right)\left[(4\text{ m})^2(1\text{ m}) - (0\text{ m})^2(1\text{ m})\right] - \left(2\text{ N/m}^2\right)\left[(1\text{ m})^2(4\text{ m}) - (1\text{ m})^2(0\text{ m})\right] = 8\text{ J}.$$



For path 2, $\mathrm{d}x = 0$, so

$$W_2 = \int_{(4,1)}^{(4,3)} \left(-axy + bx^2\right) \mathrm{d}y = \left(-\tfrac{1}{2}axy^2 + bx^2y\right)\Big|_{(4,1)}^{(4,3)}$$

$$= -\tfrac{1}{2}\left(2\text{ N/m}^2\right)\left[(4\text{ m})(3\text{ m})^2 - (4\text{ m})(1\text{ m})^2\right] + \left(2\text{ N/m}^2\right)\left[(4\text{ m})^2(3\text{ m}) - (4\text{ m})^2(1\text{ m})\right] = 32\text{ J}.$$

For path 3, $\mathrm{d}y = 0$, so

$$W_3 = \int_{(4,3)}^{(0,3)} \left(axy - by^2\right) \mathrm{d}x = \left(\tfrac{1}{2}ax^2y - by^2x\right)\Big|_{(4,3)}^{(0,3)}$$

$$= \tfrac{1}{2}\left(2\text{ N/m}^2\right)\left[(0\text{ m})^2(3\text{ m}) - (4\text{ m})^2(3\text{ m})\right] - \left(2\text{ N/m}^2\right)\left[(3\text{ m})^2(0\text{ m}) - (3\text{ m})^2(4\text{ m})\right] = 24\text{ J}.$$

For path 4, $\mathrm{d}x = 0$, so

$$W_4 = \int_{(0,3)}^{(0,1)} \left(-axy + bx^2\right) \mathrm{d}y = \left(-\tfrac{1}{2}axy^2 + bx^2y\right)\Big|_{(0,3)}^{(0,1)}$$

$$= -\tfrac{1}{2}\left(2\text{ N/m}^2\right)\left[(4\text{ m})(3\text{ m})^2 - (4\text{ m})(1\text{ m})^2\right] + \left(2\text{ N/m}^2\right)\left[(0\text{ m})^2(1\text{ m}) - (0\text{ m})^2(3\text{ m})\right] = 0\text{ J}.$$

Thus, $W_{\text{total}} = W_1 + W_2 + W_3 + W_4 = \boxed{+64\text{ J}}$.

57. For a variable force, the work is found by integrating:

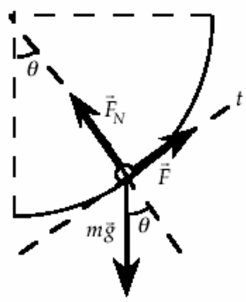
$$W_x = \int_0^x \left(F_0 + Cx'\right) \mathrm{d}x' = \left(F_0\,x' + \frac{1}{2}\,C\,x'^2\right)\Bigg|_0^x = F_0\,x + \frac{1}{2}\,Cx^2.$$

(a) $W_1 = F_0x + \tfrac{1}{2}Cx^2 = (5\text{ N})(1\text{ m}) + \tfrac{1}{2}(-2\text{ N/m})(1\text{ m})^2 = \boxed{4\text{J}}$.
$W_2 = (5\text{ N})(2\text{ m}) + \tfrac{1}{2}(-2\text{ N/m})(2\text{ m})^2 = \boxed{6\text{J}}$.
$W_3 = (5\text{ N})(3\text{ m}) + \tfrac{1}{2}(-2\text{ N/m})(3\text{ m})^2 = \boxed{6\text{J}}$.
$W_4 = (5\text{ N})(4\text{ m}) + \tfrac{1}{2}(-2\text{ N/m})(4\text{ m})^2 = \boxed{4\text{J}}$.

(b) $W = 0 = F_0x + \tfrac{1}{2}Cx^2 = x(F_0 + \tfrac{1}{2}Cx)$, which gives
$x = 0$ and $x = -2F_0/C = -2(5\text{ N})/(-2\text{ N/m}) = \boxed{+5\text{m}}$.

(c) In one dimension, a force that is a function of position only is $\boxed{\text{conservative}}$.

60. (a) Because the tangential speed is constant, we have
$\Sigma F_t = 0 = F - mg\sin\theta$, or $F = mg\sin\theta$.
Because the force is variable, we find the work by integrating over the path, with a differential tangential displacement of $ds = R\,d\theta$:



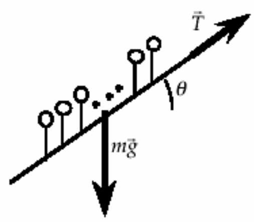
$$W_F = \int -\vec{F}\cdot d\vec{r} = \int F\,ds = \int_0^{\pi/2} (mg\sin\theta)\,R\,d\theta$$
$$= mgR\int_0^{\pi/2}\sin\theta\,d\theta = mgR(-\cos\theta)\Big]_0^{\pi/2}$$
$$= mgR[0-(-1)] = mgR.$$

(b) Because the speed does not change, from the work-energy theorem we have $W_{\text{net}} = \Delta K = 0$. The normal force is always perpendicular to the path and thus does no work, so we have
$W_{\text{net}} = W_N + W_F + W_g = 0 + W_F + W_g = 0$, or
$W_F = -W_g = -(-mg)\,\Delta h = \boxed{mgR}$.

66. (a) $W_1 = F_1 x = (0.05\text{ N})(2.5\text{m}) = \boxed{0.13\text{ J}}$.
$W_2 = F_2 x = (0.75\text{ N})(2.5\text{ m}) = \boxed{1.9\text{ J}}$.

(b) We could find either the time of the motion or the average speed. From the work-energy theorem,
$W_{\text{net}} = \Delta K$;
$0.13\text{ J} + 1.9\text{ J} = \frac{1}{2}(80\text{ kg})(v_f^2 - 0)$, which gives $v_f = 0.224\text{ m/s}$.
Because the forces are constant, the acceleration is also constant and thus
$v_{\text{av}} = \frac{1}{2}(v_i + v_f) = \frac{1}{2}(0.224\text{ m/s} + 0) = 0.112\text{ m/s}$; so
$P_1 = F_1 v_{\text{av}} = (0.05\text{ N})(0.112\text{ m/s}) = \boxed{6\times 10^{-3}\text{ W}}$ and
$P_2 = F_2 v_{\text{av}} = (0.75\text{ N})(0.112\text{ m/s}) = \boxed{8.4\times 10^{-2}\text{ W}}$.

72. Because the speed is constant, the force provided by the motor corresponds to a tension and must equal the magnitude of the component of the force of gravity along the escalator:
$T = mg\sin\theta$.
The power generated by the motor is



$P = Tv = (mg\sin\theta)v$
$= (75\text{ passengers})(75\text{ kg/passenger})(9.8\text{ m/s}^2)(\sin 20°)(1.2\text{ m/s})$
$= 2.3\times 10^4\text{ W} = 23\text{ kW} = \boxed{30\text{ hp}}$.

82. (a) $W_{F1} = \vec{F}_1\cdot\Delta\vec{r} = (2\hat{i} + 7\hat{j}\text{ N})\cdot[(10\hat{i} + 5\hat{j})\text{ m} - (0\hat{i} + 0\hat{j})\text{ m}] = \boxed{55\text{ J}}$.

(b) $P_1 = \vec{F}_1\cdot\vec{v}_i = (2\hat{i} + 7\hat{j})\text{N}\cdot(2\hat{i} + \hat{j})\text{ m/s} = \boxed{11\text{ W}}$.

(c) $W_{F2} = \vec{F}_2\cdot\Delta\vec{r} = (2\hat{i} - 5\hat{j})\text{ N}\cdot[(10\hat{i} + 5\hat{j})\text{ m} - (0\hat{i} + 0\hat{j})\text{m}] = \boxed{-5\text{ J}}$.
From the work-energy theorem,
$W_{\text{net}} = K_f - K_i$; $W_{F1} + W_{F2} = K_f - \frac{1}{2}mv_i^2$;
$55\text{ J} - 5\text{ J} = K_f - \frac{1}{2}(3\text{ kg})[(2\text{ m/s})^2 + (1\text{ m/s})^2]$, which gives $K_f = \boxed{58\text{ J}}$.

86. (a) Because the resistive force must balance the force provided by the engine, we have
$F = P/v = (45\ \text{hp})(746\ \text{W/hp})/[(80\ \text{km/h})/(3.6\ \text{ks/h})] = \boxed{1.2 \times 10^2\ \text{N}}$.

(b) If the resistive force is proportional to the velocity, the engine force is also proportional to the velocity. Thus $P = Fv = kv^2$, or $P_2 = (v_2/v_1)^2 P_1$.
$P_2 = [(60\ \text{km/h})/(80\ \text{km/h})]^2(45\ \text{hp}) = \boxed{25\ \text{hp}}$.

(c) $P_3 = (v_3/v_1)^2 P_1 = [(140\ \text{km/h})/(80\ \text{km/h})]^2(45\ \text{hp}) = \boxed{1.4 \times 10^2\ \text{hp}}$.

89. (a) Because the tension in the rope is perpendicular to the motion, $W_T = \boxed{0}$.

(b) With $F_N = mg$, we have $f_k = \mu_k mg$. The friction force is opposite to the velocity and thus tangent to the path. The work done by the friction force is
$W_f = -\mu_k mg(2\pi r) = -0.02(0.2\ \text{kg})(9.8\ \text{m/s}^2)(2\pi)(0.8\ \text{m}) = \boxed{-0.20\ \text{J}}$.

(c) $W_{\text{net}} = \Delta K$; $-0.20\ \text{J} = K_f - \frac{1}{2}(0.2\ \text{kg})(10\ \text{m/s})^2$, which gives $K_f = \boxed{9.8\ \text{J}}$.

92. (a) $P = W/\Delta t = (6 \times 10^4\ \text{J})/(0.3\text{x}10^{-9}\ \text{s}) = 2 \times 10^{14}\ \text{W} = \boxed{2 \times 10^{11}\ \text{kW}}$.

(b) $P = W/\Delta t = (6 \times 10^4\ \text{J})/[(20\ \text{min})(60\ \text{s/min})] = \boxed{50\text{W}}$.

95. If we assume no change in the kinetic energy, we have
$W_{\text{net}} = \Delta K$, or $W_{\text{body}} + W_{\text{sun}} = 0$. Thus
$W_{\text{body}} = -W_{\text{sun}}$.
We take x as positive away from the sun. Because $F(x)$ is toward the sun, we have
$W_{\text{body}} = -\int F(x)\ dx = -\int (-mK/x^2)\ dx = +\int (mK/x^2)\ dx$.
For the small variation of $\Delta x = 1\%$ of x, we can take the force to be constant and get
$W_{\text{body}} \approx -(-mK/x^2)\int dx = +(mK/x^2)(0.01x) = \boxed{0.01mK/x}$.