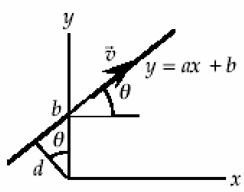
6. Because the particle is traveling in a straight line, $y = ax + b$, its angular momentum about the origin will be constant. From the diagram the right-hand rule gives its direction as into the page, or $-\hat{k}$. We find the angle θ from $\tan\theta = dy/dx = a$, so $\cos\theta = 1/(1 + a^2)^{1/2}$. The magnitude of the angular momentum is

$$L = pd = mvb\cos\theta = mbv[1/(1 + a^2)^{1/2}].$$ Thus

$$\vec{L} = \boxed{-[mbv/\sqrt{1 + a^2}\,]\hat{k}}.$$



11. The location of the center of mass is defined by $\vec{R} = (m_1\vec{r}_1 + m_2\vec{r}_2)/(m_1 + m_2)$. If $\vec{R} = 0$, we have

$$\vec{r}_1 = -(m_2/m_1)\vec{r}_2.$$

If we let $\vec{r} = \vec{r}_2 - \vec{r}_1$, we get

$$\vec{r}_2 - \vec{r} = -(m_2/m_1)\vec{r}_2,$$ which reduces to $\vec{r}_2 = m_1\vec{r}/(m_1 + m_2)$.

For the momenta, we have

$$\vec{p}_1 = m_1\vec{v}_1 = m_1\,d\vec{r}_1/dt \text{ and } \vec{p}_2 = m_2\vec{v}_2 = m_2\,d\vec{r}_2/dt.$$

That the center of mass is at rest means that $\vec{P} = \vec{p}_1 + \vec{p}_2 = 0$, or $\vec{p}_1 = -\vec{p}_2$.

Using this result, the total angular momentum of the system is

$$\vec{L} = (\vec{r}_1 \times \vec{p}_1) + (\vec{r}_2 \times \vec{p}_2) = (\vec{r}_2 - \vec{r}_1) \times \vec{p}_2,$$ which we can write as

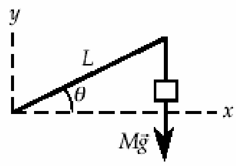
$$\vec{L} = (\vec{r}_2 - \vec{r}_1) \times (m_2\vec{v}_2) = (\vec{r}_2 - \vec{r}_1) \times (m_2\,d\vec{r}_2/dt).$$ In terms of $\vec{r}$, this becomes

$$\vec{L} = \boxed{\vec{r} \times [m_2m_1/(m_1 + m_2)]\,d\vec{r}/dt = \vec{r} \times \mu\,d\vec{r}/dt}.$$

This is the angular momentum of a particle of mass μ at a position $\vec{r}$.

18. With the coordinate system shown, we find the torque from

$$\vec{\tau} = \vec{r} \times \vec{F}$$
$$= (L\cos\theta\,\hat{i} + L\sin\theta\,\hat{j}) \times (-Mg\,\hat{j})$$
$$= -MgL\cos\theta\,\hat{k}$$
$$= -(18\text{ kg})(9.8\text{ m/s}^2)(2.2\text{ m})(\cos 20°)\,\hat{k}$$
$$= \boxed{-(3.6 \times 10^3\text{ N}\cdot\text{m})\hat{k}\ \ \text{(into page)}}.$$



20. For the resultant force, we have

$$\sum\vec{F} = \vec{F} + (-\vec{F}) = 0.$$

For the resultant torque, we have

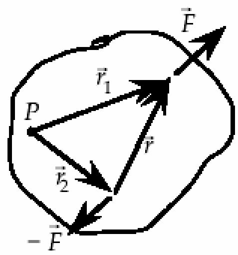
$$\vec{\tau} = \vec{r}_1 \times \vec{F} + \vec{r}_2 \times (-\vec{F}) = (\vec{r}_1 - \vec{r}_2) \times \vec{F}.$$

From the diagram, we see that

$$\vec{r}_2 + \vec{r} = \vec{r}_1, \text{ or } \vec{r}_1 - \vec{r}_2 = \vec{r}.$$

Thus the torque is

$\vec{\tau} = \vec{r} \times \vec{F}$, independent of the location of the point P.



23. The perpendicular distance from the axis A to the initial path of the ball is

$$r_i = d\sin[2(\pi/2 - \theta)] = d\sin(\pi - 2\theta) = d\sin(2\theta).$$

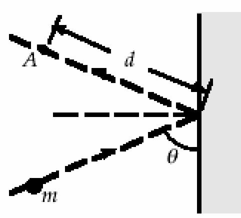
The initial angular momentum is

$$L_i = r_i mv = \boxed{mvd\sin(2\theta)\ \ \text{up}}.$$

Because the final velocity passes through the axis, we have

$$L_f = \boxed{0}.$$

The wall exerts an impulsive force on the ball and thus an **impulsive torque** that changes the angular momentum.



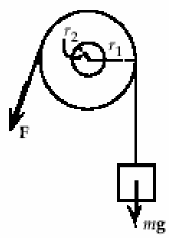
24. At a steady speed, $d\vec{L}/dt = 0$, so we have $\vec{\tau}_{net} = 0$. Both torques are along the axle, with that due to F out and that due to mg in, which we take as positive. Thus we have

$\tau_{net} = 0;$

$mgr_1 - Fr_2 = 0$, which gives

$F = mg(r_1/r_2)$

$= (200\text{ kg})(9.8\text{ m/s}^2)(10\text{ cm})/(50\text{ cm}) = \boxed{3.9 \times 10^2\text{ N}}.$



32. We use energy conservation to find the speed of the point mass before it strikes the bar:

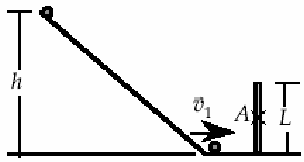
$v_1 = (2gh)^{1/2} = [2(9.8\text{m/s}^2)(1.1\text{m})]^{1/2} = 4.64\text{ m/s}.$

From the conservation of angular momentum of the system of mass and bar about the pivot point A during the collision, we have

$L_A = mv_1(\frac{1}{2}L) = [(1/12)ML^2 + m(\frac{1}{2}L)^2]\omega;$

$(0.017\text{ kg})(4.64\text{ m/s})(0.10\text{ m}) = [(1/12)(0.2\text{ kg})(0.2\text{ m})^2 + (0.017\text{ kg})(0.1\text{ m})^2]\omega,$ which gives

$\omega = \boxed{9.4\text{ rad/s}}.$



33. Label the clay 1 and the wheel 2. The initial speed of the clay just before hitting the wheel is

$v_1 = (2gh)^{1/2} = [2(9.8\text{ m/s}^2)(0.75\text{ m})]^{1/2} = 3.834\text{ m/s}.$

Relative to the center of the wheel the angular momentum of the clay-wheel system just before their collision is

$L_i = m_1 v_1 R$, with R the radius of the wheel.

After the collision the rotational inertia of the system is $I_f = m_1 R^2 + \frac{1}{2}m_2 R^2$, and so the angular speed ω of the system is obtained from conservation of angular momentum:

$L_i = m_1 v_1 R = L_f = I_f\omega = (m_1 R^2 + \frac{1}{2}m_2 R^2)\omega$, or

$\omega = m_1 v_1/[(m_1 + \frac{1}{2}m_2)R]$

$= (0.100\text{ kg})(3.834\text{ m/s})/[(0.100\text{ kg} + 10\text{ kg}/2)(0.50\text{ m})]$

$= \boxed{0.15\text{ rad/s}}.$

36. We choose the reference level for gravitational potential energy at the initial position. The kinetic energy will be the translational energy of the center of mass and the rotational energy about the center of mass. Because there is no work done by friction while the cylinder is rolling, for the work-energy theorem we have

$W_{net} = \Delta K + \Delta U;$

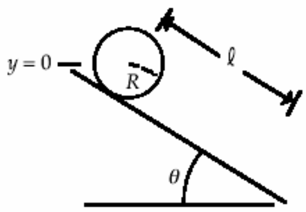
$0 = (\frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 - 0) + Mg(0 - \ell\sin\theta).$

Because the cylinder is rolling, $v = R\omega$. The rotational inertia is $\frac{1}{2}MR^2$. Thus we get

$\frac{1}{2}Mv^2 + \frac{1}{2}(\frac{1}{2}MR^2)(v^2/R^2) = Mg\ell\sin\theta$, which gives

$v = (4g\ell\sin\theta/3)^{1/2}$, and $\omega = (4g\ell\sin\theta/3)^{1/2}/R.$

We obtain the same result as in Section 9–7, but more directly.



38. The component of the pulling force $\vec{F}$ that is parallel to the direction of motion of the roller is $F\cos\theta$, where $F = 55$ N and $\theta = 45°$. The work done by that force after it pulls the roller through a distance d is $W = Fd\cos\theta$, which result in a change in the kinetic energy of the roller:

$W = Fd\cos\theta = \Delta K = \frac{1}{2}MV_{cm}^2 + \frac{1}{2}I\omega^2.$

But for pure rolling $\omega = V_{cm}/R$, and for a solid cylinder $I = \frac{1}{2}MR^2$; so

$I\omega^2 = \frac{1}{2}MR^2(V_{cm}/R)^2 = \frac{1}{2}MV_{cm}^2$, whereupon

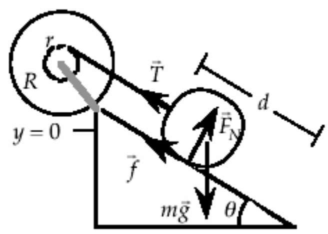
$\Delta K = \frac{3}{4}MV_{cm}^2$. Equate this with W to obtain

$V_{cm} = (\frac{4}{3}Fd\cos\theta/M)^{1/2}$

$= [\frac{4}{3}(55\text{ N})(2\text{ m})(\cos 30°)/(150\text{ kg})]^{1/2} = \boxed{0.92\text{ m/s}}.$

50. We find an expression for the speed of the block after it travels a distance d down the plane, starting from rest:

$$v^2 = v_0^2 + 2ax = 0 + 2ad = 2(0.1 \text{ m/s}^2)d = (0.2 \text{ m/s}^2)d.$$



Because the speed of the thread is the tangential speed of the axle, for the angular speed of the cylinder we have $\omega = v/r$, or

$$\omega^2 = v^2/r^2 = (0.2 \text{ m/s}^2)d/(0.005 \text{ m})^2 = (8.0 \times 10^3 \text{ rad}^2/\text{m}\cdot\text{s}^2)d.$$

The rotational inertia of the cylinder is

$$I = \tfrac{1}{2}MR^2 = \tfrac{1}{2}(0.5 \text{ kg})(0.04 \text{ m})^2 = 4.0 \times 10^{-4} \text{ kg}\cdot\text{m}^2.$$

We choose the reference level for gravitational potential energy at the initial position of the mass.

For the work-energy theorem applied to the system of cylinder and mass, we have

$$W = \Delta K + \Delta U;$$

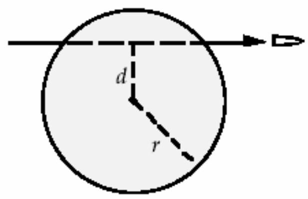
$$-\mu_k mg \cos\theta\, d = (\tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 - 0) + mg(0 - d\sin\theta);$$

$$-\mu_k(1 \text{ kg})(9.8 \text{ m/s}^2)\cos 30°\, d =$$

$$\tfrac{1}{2}(1 \text{ kg})(0.2 \text{ m/s}^2)d + \tfrac{1}{2}(4.0 \times 10^{-4} \text{ kg}\cdot\text{m}^2)(8.0 \times 10^3 \text{ rad}^2/\text{m}\cdot\text{s}^2)d - (1 \text{ kg})(9.8 \text{ m/s}^2)\sin 30°\, d.$$

Note that the distance d cancels out, so we get $\mu_k = \boxed{0.38}$.

54. The bullet passes through the wheel so quickly that it leaves before the wheel turns. For this collision, angular momentum of the bullet-wheel system is conserved:



$$L = mv_i d = mv_f d + I\omega, \text{ which we write as}$$

$$\tfrac{1}{2}MR^2\omega = ma(v_i - v_f);$$

$$\tfrac{1}{2}(3.0 \text{ kg})(0.18 \text{ m})^2\omega = (1.5 \times 10^{-2} \text{ kg})(0.14 \text{ m})(350 \text{ m/s} - 270 \text{ m/s}),$$

which gives

$$\omega = \boxed{3.4 \text{ rad/s (clockwise)}}.$$

The angular momentum of the wheel is

$$L = \tfrac{1}{2}MR^2\omega = \tfrac{1}{2}(2.0 \text{ kg})(0.18 \text{ m})^2(3.4 \text{ rad/s})$$

$$= \boxed{0.17 \text{ kg}\cdot\text{m}^2/\text{s (clockwise)}}.$$

The kinetic energy of the wheel is

$$K = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}[\tfrac{1}{2}(2.0 \text{ kg})(0.18 \text{ m})^2](3.4 \text{ rad/s})^2 = \boxed{0.28 \text{ J}}.$$

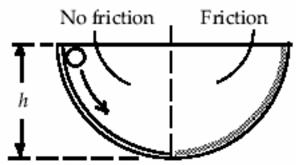
The change in the kinetic energy of the bullet is

$$\Delta K_{\text{bullet}} = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_0^2 = \tfrac{1}{2}(1.5 \times 10^{-2} \text{ kg})[(270 \text{ m/s})^2 - (350 \text{ m/s})^2]$$

$$= \boxed{-3.7 \times 10^2 \text{ J}}.$$

The total kinetic energy is not conserved. Negative work is done by the drag forces as the bullet passes through the wheel.

57. While the ball is sliding down the smooth side of the bowl, the kinetic energy of translation increases as the potential energy decreases. If the ball starts rolling immediately at the bottom, the distance in which sliding changes to rolling is very short and the work done by friction can be neglected. There will be both translational and rotational kinetic energy as the ball starts rolling up the other side. While the ball is rolling up the side, friction does no work. Thus we use the work-energy theorem from the release point to the point where the ball momentarily comes to rest:



$$W_{\text{net}} = \Delta K + \Delta U;$$

$$0 = (0 - 0) + Mg(h' - h), \text{ which gives}$$

$$\boxed{h' = h}.$$

With our assumption of no work done by the friction forces, the initial and final potential energies must be the same.

58. (a) Each horizontal section of the door can be considered to be a rod, so we have

$I_d = \frac{1}{3}M\ell^2 = \frac{1}{3}(15\ \text{kg})(1.2\ \text{m})^2 = \boxed{7.2\ \text{kg}\cdot\text{m}^2}.$

(b) During the collision, angular momentum of the door-mud ball system about an axis through the hinges is conserved:

$L = mv\ell = (I_d + m\ell^2)\omega;$

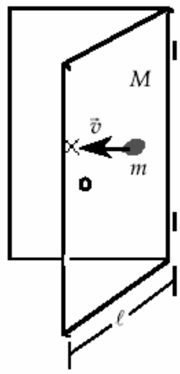
$(0.3\ \text{kg})(12\ \text{m/s})(1.2\ \text{m}) = [7.2\ \text{kg}\cdot\text{m}^2 + (0.3\ \text{kg})(1.2\ \text{m})^2]\omega,$ which gives

$\omega = \boxed{0.57\ \text{rad/s}}.$

(c) $K_f/K_i = \frac{1}{2}(I_d + m\ell^2)\omega^2/\frac{1}{2}mv^2$

$= [7.2\ \text{kg}\cdot\text{m}^2 + (0.3\ \text{kg})(1.2\ \text{m})^2](0.566\ \text{rad/s})^2/[(0.3\ \text{kg})(12\ \text{m/s})^2]$

$= \boxed{0.057}.$



68. For the inelastic collision, angular momentum about the contact point A is conserved:

$L_A = mv\ell = (\frac{1}{3}M\ell^2 + m\ell^2)\omega_0,$

which gives the initial angular speed for the falling;

$\omega_0 = 3mv/[(M + 3m)\ell] = 3mv/[(5m + 3m)\ell] = 3v/8\ell.$

After the collision, the rotational inertia about the contact point is

$I_A = \frac{1}{3}M\ell^2 + m\ell^2 = 8m\ell^2/3.$

If the contact point does not move, no work is done by the friction force. For the falling motion, we use energy conservation, with the reference level for potential energy at the table:

$K_i + U_i = K_f + U_f;$

$\frac{1}{2}I_A\omega_0^2 + (\frac{1}{2}Mg\ell) + mg\ell = \frac{1}{2}I_A\omega^2 + 0$

$\frac{1}{2}(8m\ell^2/3)(3v/8\ell)^2 + \frac{1}{2}(5m)g\ell + mg\ell = \frac{1}{2}(8m\ell^2/3)\omega^2 + 0,$

which gives $\omega = \boxed{(9v^2/16 + 21g\ell/2)^{1/2}/2\ell,\ \text{clockwise}}.$

The angular momentum as the rod hits is

$L = I\omega = (8m\ell^2/3)(9v^2/16 + 21g\ell/2)^{1/2}/2\ell$

$= \boxed{(4m\ell/3)(9v^2/16 + 21g\ell/2)^{1/2},\ \text{clockwise}}.$

The kinetic energy is

$K = \frac{1}{2}I\omega^2 = \boxed{m\ell^2(3v^2/4 + 14g\ell)}.$