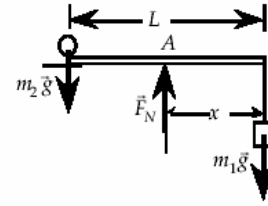


9. We choose the coordinate system shown, with positive torques clockwise. As the plank is moved out from the roof, the effective normal force acts at a point closer to the edge. When the normal force reaches the edge, the plank is on the verge of tipping. We write  $\Sigma \tau = I\alpha$  about the edge  $A$  from the force diagram for the plank, the load and the concrete:

$$\Sigma \tau_A = M_1 g x - M_2 g (L - x) = 0, \text{ which gives}$$

$$x = M_2 L / (M_1 + M_2) = (15 \text{ kg})(2.4 \text{ m}) / (30 \text{ kg} + 15 \text{ kg}) = \boxed{0.8 \text{ m}}.$$



19. We choose the coordinate system shown, with positive torques counterclockwise.

- (a) We write  $\Sigma \tau = I\alpha$  about the point  $B$  from the force diagram for the ladder and man:

$$\Sigma \tau_B = mg(\frac{1}{2}L) \sin \theta + Mg d \sin \theta - F_{N\text{top}} L \cos \theta = 0.$$

We write  $\Sigma F_x = ma_x$  from the force diagram for the ladder and man:

$$F_{N\text{top}} - f_{\text{bottom}} = 0.$$

We write  $\Sigma F_y = ma_y$  from the force diagram for the ladder and man:

$$F_{N\text{bottom}} - (m + M)g = 0.$$

As the man climbs the ladder, the friction force at the bottom increases. At the point where slipping begins,

$$f_{\text{bottom}} = f_{\text{bottom,max}} = \mu_s F_{N\text{bottom}} = \mu_s (m + M)g. \text{ Then we have}$$

$$F_{N\text{top}} = \mu_s (m + M)g.$$

Using these in the torque equation, we get

$$d = [\mu_s (m + M)L \cos \theta - m(\frac{1}{2}L) \sin \theta] / (M \sin \theta)$$

$$= [0.40(10 \text{ kg} + 80 \text{ kg})(4 \text{ m}) \cos 30^\circ - 10(4 \text{ m}/2) \sin 30^\circ] / [(80 \text{ kg}) \sin 30^\circ] = \boxed{2.9 \text{ m}}.$$

- (b) We write  $\Sigma \tau = I\alpha$  about the point  $A$  from the force diagram for the ladder and man:

$$\Sigma \tau_A = F_{N\text{bottom}}(L \sin \theta) - Mg(L - d) \sin \theta - mg(\frac{1}{2}L) \sin \theta - f_{\text{bottom}} L \cos \theta = 0.$$

We write  $\Sigma \tau = I\alpha$  about the point  $O$  from the force diagram for the ladder and man:

$$\Sigma \tau_O = F_{N\text{bottom}}(L \sin \theta) - Mg(L - d) \sin \theta - mg(\frac{1}{2}L) \sin \theta - F_{N\text{top}} L \cos \theta = 0.$$

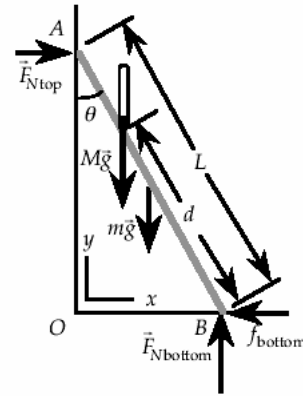
We can combine these three torque equations to obtain the force equations.

If we add the equations for points  $A$  and  $O$ , we get

$$F_{N\text{top}} = f_{\text{bottom}}, \text{ which is the } x\text{-equation.}$$

If we subtract the equations for points  $B$  and  $O$ , we get

$$F_{N\text{bottom}} = \mu_s (m + M)g, \text{ which is the } y\text{-equation.}$$



27. The force  $F$  supplies a counterclockwise torque about the axis of the pulleys,  $\tau_1 = FR_1$ , where  $R_1 = 20 \text{ cm}$ ; while the weight of the engine (of mass  $m$ ) exerts a clockwise torque about the axis,  $\tau_2 = -mgR_2$ , where  $R_2 = 8.0 \text{ cm}$ . To balance the torques, set

$$\Sigma \tau_A = FR_1 - mgR_2 = 0, \text{ which gives}$$

$$F = mgR_2 / R_1 = (300 \text{ kg})(9.8 \text{ m/s}^2)(8.0 \text{ cm}) / 20 \text{ cm} = \boxed{1.2 \text{ kN}}.$$

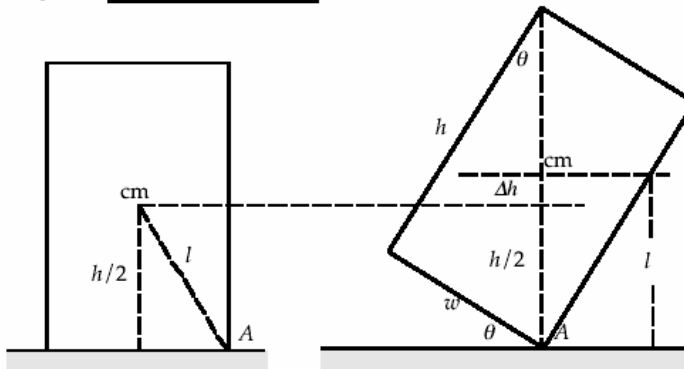
38. See the diagram below. The block must be brought to such a position that its center of mass (point  $O$ ) is directly above the axis of rotation (through point  $A$ ). The critical angle in question is

$$\theta = \boxed{\tan^{-1}(w/h)}.$$

The work  $W$  done must be at least equal to the increase in gravitational potential energy of the block (assuming that the block virtually stops at the critical position). Since the center of mass rises by

$$\Delta h = l - \frac{1}{2}h = [(\frac{1}{2}h)^2 + (\frac{1}{2}w)^2]^{1/2} - \frac{1}{2}h,$$

$$W_{\text{min}} = mg \Delta h = \boxed{\frac{1}{2}mg [(h^2 + w^2)^{1/2} - h]}.$$



42. We choose the coordinate system shown, with positive torques clockwise.

(a) We write  $\sum \tau = I\alpha$  about the point A from the force diagram for the rock:

$$\sum \tau_A = F_N R \sin \theta - f_s (R + R \cos \theta) = 0, \text{ which gives}$$

$$f_s = [\sin \theta / (1 + \cos \theta)] F_N = \tan(\frac{1}{2}\theta) F_N.$$

The larger the angle becomes the greater the friction force required. The maximum friction force is  $f_{s\max} = \mu_s F_N$ , so we find the maximum angle from

$$\tan(\frac{1}{2}\theta_{\max}) = f_{s\max} / F_N = \mu_s = 0.6, \text{ which gives}$$

$$\theta_{\max} = \boxed{62^\circ}.$$

(b) We write  $\sum \tau = I\alpha$  about the point B from the force diagram for the rock:

$$\sum \tau_B = T(R + R \cos \theta) - mgR \sin \theta = 0, \text{ which gives}$$

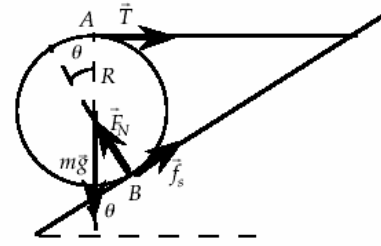
$$T_{\max} = [\sin \theta_{\max} / (1 + \cos \theta_{\max})] mg$$

$$= \tan(\frac{1}{2}\theta_{\max}) mg = \mu_s mg$$

$$= (0.6)(1088 \text{ kg})(9.8 \text{ m/s}^2) = \boxed{6 \times 10^3 \text{ N}}.$$

(c) When the angle is less than the maximum angle, from part (b) we get

$$T = \boxed{mg \tan(\frac{1}{2}\theta)}.$$



55. We call the length of the beam  $L_{\text{beam}}$ . The initial length of the cable is  $L_0 = L_{\text{beam}} / \cos 30^\circ = 2L_{\text{beam}} / \sqrt{3}$  and the distance along the wall from the beam to the cable is  $D = L_{\text{beam}} \tan 30^\circ = L_{\text{beam}} / \sqrt{3}$ . When the load is hung at the end of the beam, there must be an additional tension in the cable to maintain equilibrium:

$$\sum \tau_A = (\Delta T \sin 30^\circ) L_{\text{beam}} - Mg L_{\text{beam}} = 0, \text{ or}$$

$$\Delta T = Mg / \sin 30^\circ = (30 \text{ kg})(9.8 \text{ m/s}^2) / \sin 30^\circ = 588 \text{ N},$$

which is an additional stress:

$$\Delta T / \pi r^2 = 588 \text{ N} / [\pi(1 \times 10^{-3} \text{ m})^2] = 1.87 \times 10^8 \text{ N/m}^2.$$

(Note that even when the initial tension is added, this is less than the tensile strength, so the cable does not break.)

The additional stress produces an elongation of the cable:

$$\Delta L / L_0 = \Delta T / AY.$$

This elongation causes the beam to drop below the horizontal (exaggerated in the diagram). We find the angle  $\theta$  from a geometrical formula for a triangle:

$$D^2 + L_{\text{beam}}^2 - 2DL_{\text{beam}} \cos \theta = (L_0 + \Delta L)^2 = L_0^2(1 + \Delta L / L_0)^2 \approx L_0^2(1 + 2\Delta L / L_0),$$

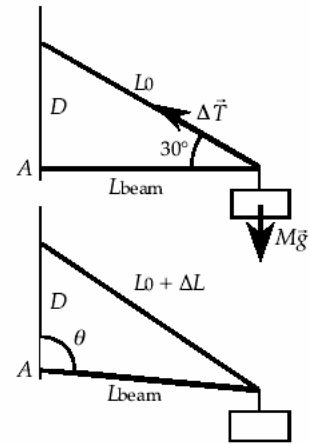
where we have used the fact that  $\Delta L \ll L_0$ . Because  $D^2 + L_{\text{beam}}^2 = L_0^2$ , this becomes

$$-2(L_{\text{beam}} / \sqrt{3})L_{\text{beam}} \cos \theta = (2L_{\text{beam}} / \sqrt{3})^2 2\Delta L / L_0, \text{ which reduces to}$$

$$\cos \theta = -(4/\sqrt{3})(\Delta L / L_0) = -(4/\sqrt{3})(\Delta T / AY)$$

$$= (4/\sqrt{3})(588 \text{ N}) / [\pi(1 \times 10^{-3} \text{ m})^2(2.1 \times 10^5 \text{ MN/m}^2)] = 0.00206, \text{ which gives } \theta = 90.12^\circ.$$

Thus the beam is  $\boxed{112^\circ}$  below the horizontal.



59. (a) We have an induced transverse strain, given by  $e_{\text{tr}} = -\alpha e_1$ . The total strain on a side of the cube will be due to the direct strain and the induced transverse strain from the other two sides:

$$e = \Delta L / L = e_1 + 2e_{\text{tr}} = e_1(1 - 2\alpha).$$

The volume of the cube is  $V = L^3$ , so a small change in  $L$  means a small change in  $V$ , given by

$$\Delta V = 3L^2 \Delta L, \text{ which gives a volume strain:}$$

$$\Delta V / V = 3 \Delta L / L = 3e_1(1 - 2\alpha).$$

If we use the relation between stress and strain, we have

$$\Delta V / V = 3[(F/A)/Y](1 - 2\alpha).$$

All three sides are under pressure, which is the stress, so we have for the magnitude

$$\Delta V / V = 3p(1 - 2\alpha) / Y.$$

(b) The bulk modulus is defined as

$$B = -p / (\Delta V / V).$$

If we substitute the given volume change, which must be negative for a positive pressure, we get

$$B = -p / [-3p(1 - 2\alpha) / Y] = Y / [3(1 - 2\alpha)].$$

65. We choose the coordinate system shown, with positive torques counterclockwise.

(a) We write  $\Sigma \tau = I\alpha$  about the point A from the force diagram for the beam:

$$\Sigma \tau_A = Mg(\frac{1}{2}L) \cos \theta_0 - TL \sin \theta_0 = 0, \text{ which gives } T = \frac{1}{2}Mg \cot \theta_0$$

(b) When the cable snaps, the tension is zero, so we have

$$\Sigma \tau_A = I\alpha;$$

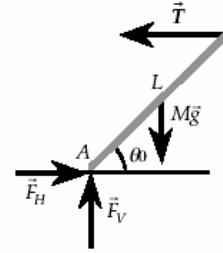
$$Mg(\frac{1}{2}L) \cos \theta_0 = \frac{1}{3}ML^2\alpha, \text{ which gives } \alpha = \frac{(3g \cos \theta_0)}{2L}$$

(c) The angular acceleration is not constant. Rather than integrate, we use the work-energy theorem, with the reference level for potential energy at the horizontal position. No work is done by the force at the pivot, so we have

$$W = \Delta K + \Delta U$$

$$0 = (\frac{1}{2}I_A\omega^2 - 0) + Mg(0 - \frac{1}{2}L \sin \theta_0).$$

When we use  $I_A = \frac{1}{3}ML^2$ , we get  $\omega = \sqrt{(3g \sin \theta_0)/L}$ .



75. We choose the coordinate system shown, with positive torques clockwise. We write  $\Sigma \tau = I\alpha$  about the point A from the force diagram for the cylinder:

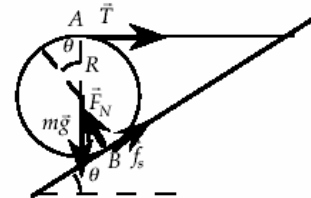
$$\Sigma \tau_A = F_N R \sin \theta - f_s (R + R \cos \theta) = 0, \text{ which gives}$$

$$f_s = [\sin \theta / (1 + \cos \theta)] F_N = \tan(\frac{1}{2}\theta) F_N.$$

Because  $f_s \leq \mu_s F_N$ , the minimum value of  $\mu_s$  is when  $f_s = \mu_s F_N$ . So

$$\mu_{s\min} F_N = \tan(\frac{1}{2}\theta) F_N, \text{ which gives}$$

$$\mu_{s\min} = \tan(\frac{1}{2}\theta)$$



77. The net force exerted on the ball is zero, so

$$F_{NA} - T \sin \theta = 0 \text{ in the horizontal direction and}$$

$$f + T \cos \theta - mg = 0 \text{ in the vertical direction.}$$

Also, the net torque about point A must vanish. The lever arm of T about point A is  $AD = CA \sin \theta = CB \sin \theta$ , and that of the weight of the ball is R, its radius. Here  $\theta = \sin^{-1}(11 \text{ cm} / 90 \text{ cm}) = 7.02^\circ$ . Thus

$$\Sigma \tau_A = T(CB \sin \theta) - mgR = 0.$$

And when  $\mu$  is at its smallest possible value

$$f = f_{\max} = \mu F_{NA}. \text{ Combine these equation to obtain}$$

$$\mu = CB/R - \cot \theta = 90 \text{ cm} / 6.0 \text{ cm} - \cot 7.02^\circ = \boxed{6.88}.$$

