15. With the reference level for potential energy at the ground, we use energy conservation to relate the maximum height to the initial speed:

$K_i + U_i = K_f + U_f$;

$\frac{1}{2}mv_0{}^2 + 0 = 0 + mgh$, which gives $v_0{}^2 = 2gh$.

Because we assume that the initial speed is constant, with g_{Mars} from Problem 8, we have

$g_{\text{Mars}} h_{\text{Mars}} = g_E h_E$, or

$h_{\text{Mars}} = (g_E/g_{\text{Mars}})h_E = [(9.8\ \text{m/s}^2)/(3.70\ \text{m/s}^2)](1.85\ \text{m}) = \boxed{4.9\ \text{m}}$.

26. We use conservation of energy, with the reference level for potential energy at infinity:

$K_i + U_i = K_f + U_f$;

$\frac{1}{2}mv_0{}^2 - GMm/R = 0 + 0$, which gives

$\frac{1}{2}mv_0{}^2 = GMm/R = (6.67 \times 10^{-11}\ \text{N·m}^2/\text{kg}^2)(7.35 \times 10^{22}\ \text{kg})(3800\ \text{kg})/(1.74 \times 10^6\ \text{m}) = \boxed{1.07 \times 10^{10}\ \text{J}}$.

31. From Kepler's third law, we have

$T^2 = 4\pi^2R^3/GM$

$= 4\pi^2[(3393 + 95) \times 10^3\ \text{m}]^3/[(6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2)(6.42 \times 10^{23}\ \text{kg})]$, which gives

$T = 6.25 \times 10^3\ \text{s} = \boxed{1.74\ \text{h}}$.

37. For the conservation of angular momentum, we have

$L = mv\,(\cos 45°)\,R = mv_f r$, which gives $v = (r/R)\,v_f/\cos 45°$.

For the conservation of energy, we have

$K_i + U_i = K_f + U_f$;

$\frac{1}{2}mv^2 - GMm/R = \frac{1}{2}mv_f^2 - GMm/r$;

Using the escape speed, given by $v_{\text{esc}}{}^2 = 2GM/R$, and the result from angular momentum conservation, with $r = 2R$, we get

$[(r/R)\,v_f/\cos 45°]^2 - v_{\text{esc}}{}^2 = v_f^2 - v_{\text{esc}}{}^2(R/r)$;

$[2v_f/\cos 45°]^2 - v_{\text{esc}}{}^2 = v_f^2 - \frac{1}{2}v_{\text{esc}}{}^2$, which reduces to

$v_f^2 = v_{\text{esc}}{}^2/14 = (11.2\ \text{km/s})^2/14$, which gives

$v_f = \boxed{2.99\ \text{km/s}}$.

40. With v_1 the initial speed, before the firing, the angular momentum is

$L_1 = mv_1R_1$.

In the circular orbit, for Newton's second law, we have

$GMm/R_1{}^2 = mv_1{}^2/R_1$, or $v_1{}^2 = GM/R_1$.

If the firing does not change the total energy, the speed immediately after firing is still v_1, since there has been no change in the position and thus no change in the potential energy.

The firing changed the direction of the satellite to decrease the angular momentum:

$L_2 = mv_1R_1 \cos\theta = \frac{1}{2}mv_1R_1$.

At apogee and perigee, the velocity is perpendicular to the radius, so the angular momentum is

$L_2 = mvr = \frac{1}{2}mv_1R_1$, which gives $v = \frac{1}{2}(R_1/r)v_1$.

Using the result for $v_1{}^2$, we can write this as $v^2 = (GM/4R_1)(R_1/r)^2$.

For the conservation of energy, we have

$K_1 + U_1 = K + U$;

$\frac{1}{2}mv_1{}^2 - GMm/R_1 = \frac{1}{2}mv^2 - GMm/r$,

$\frac{1}{2}GM/R_1 - GM/R_1 = \frac{1}{2}(GM/4R_1)(R_1/r)^2 - GMm/r$, which reduces to

$(R_1/r)^2 - 8(R_1/r) + 4 = 0$.

When we solve this quadratic equation, we get

$R_1/r = 7.46$ and 0.336, which gives

$r = \boxed{0.134R_1,\ 1.86R_1}$.

45. For the approximate form, we have

$$g(h)/g(0) \approx 1 - 2h/R_E = 1 - 2(10 \times 10^3\ \text{m})/(6.37 \times 10^6\ \text{m}) = \boxed{0.996860}.$$

For the exact form, we have

$$g(h)/g(0) = R_E^2/(R_E + h)^2 = (6.37 \times 10^6\ \text{m})^2/[(6.37 + 0.01) \times 10^6\ \text{m}]^2 = \boxed{0.996868}.$$

48. From Example 12-9, the attractive force on a mass m inside the sphere is

$$F = -GM_{\text{inside}}m/r^2 = -\tfrac{4}{3}\pi G m \rho r = -GmMr/R^3.$$

To lift the mass, we apply a force opposite to this. We integrate to find the work:

$$W = \int_{R/2}^{R} \vec{F} \cdot d\vec{r} = \int_{R/2}^{R} \frac{GmM}{R^3}\, r\, dr = \frac{GmM}{R^3}\left[\tfrac{1}{2}(R)^2 - \tfrac{1}{2}\left(\frac{R}{2}\right)^2\right] = \frac{3}{8}\frac{GmM}{R}$$

$$= \frac{3}{8}\frac{(1\ \text{kg})(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1\ \text{kg})(5.98 \times 10^{24}\ \text{kg})}{6.37 \times 10^6\ \text{m}} = 2.36 \times 10^7\ \text{J}.$$

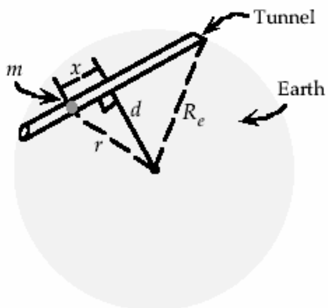
52. (a) From Example 12–9, we have $F = -\tfrac{4}{3}\pi G m \rho r$. Because the density is $\rho = M/(\tfrac{4}{3}\pi R^3)$, this becomes

$$F = -(GMm/R^3)r.$$

Because this force is radial, we can use the definition of potential energy from Section 7–2, with the origin as our reference point and $U(0) = 0$:

$$U = U(0) - \int_0^r \vec{F} \cdot d\vec{r}' = -\int_0^r -(GMm/R^3)\, r'\, dr', \text{ which gives}$$

$$U = \boxed{GMmr^2/2R^3,\ U = 0 \text{ at } r = 0}.$$



(b) In terms of x, the result from part (a) is

$$U = GMm(x^2 + d^2)/2R^3.$$

To change our reference level, we can add a constant C and have $U(x = 0) = 0$:

$$U = GMm(x^2 + d^2)/2R^3 + C;$$

$$0 = (GMmd^2/2R^3) + C, \text{ which gives } C = -GMmd^2/2R^3, \text{ and}$$

$$U = \boxed{GMmx^2/2R^3, \quad \text{with } U = 0 \text{ at } x = 0}.$$

We could also integrate the component of the force along the tunnel to get the same result.

63. (a) For a "weightless" circular orbit, the gravitational force provides the centripetal acceleration:

$$GM/R^2 = \tfrac{4}{3}\pi G m \rho R = m v_0^2/R, \text{ or}$$

$$R^2 = 3v_0^2/4\pi G\rho = 3(2.0\ \text{m/s})^2/[4\pi(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.2 \times 10^3\ \text{kg/m}^3)], \text{ which gives}$$

$$R = \boxed{1.66 \times 10^3\ \text{m}}.$$

(b) The escape speed is

$$v_{\text{esc}} = (2GM/R)^{1/2} = v_0\sqrt{2} = (2.0\ \text{m/s})\sqrt{2} = \boxed{2.8\ \text{m/s}}.$$

(c) The surface speed at the equator is

$$v = 2\pi R/T = 2\pi(1.66 \times 10^3\ \text{m})/[(12\ \text{h})(3600\ \text{s/h})] = 0.24\ \text{m/s}.$$

By walking in the direction of the rotation, he would need a speed of $\boxed{1.76\ \text{m/s}}$ to orbit the asteroid.

65. (a) For a circular orbit, we must have

$$F = k/r^n = ma = mv^2/r, \text{ which will be satisfied for a speed of } v = \sqrt{k/mr^{n-1}}.$$

$\boxed{\text{Circular orbits are supported}}$.

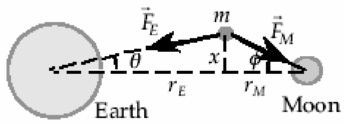
(b) The period of the circular motion is

$$T = 2\pi r/v = 2\pi r/\sqrt{k/mr^{n-1}}, \text{ which we write as}$$

$$\boxed{T^2/r^{n+1} = 4\pi^2(m/k) = \text{a constant}}.$$

70. At the equilibrium point, we have

$$GM_E m/r_E^2 = GM_M m/r_M^2.$$



When the mass is displaced by x, the two forces become

$$F_E = GM_E m/r_1^2 = GM_E m/(r_E^2 + x^2) \text{ and}$$

$$F_M = GM_M m/r_2^2 = GM_M m/(r_M^2 + x^2),$$

with the directions indicated on the diagram.

When we add the components along the line joining Earth and the Moon, we get

$$F_{||} = -F_E \cos\theta + F_M \cos\phi = -GM_E m r_E/(r_E^2 + x^2)^{3/2} + GM_M m r_M/(r_M^2 + x^2)^{3/2}.$$

Using the approximation $(r^2 + x^2)^n \approx r^{2n}(1 + nx^2/r^2)$ with $n = -\frac{3}{2}$, we get

$$F_{||} = -(GM_E m/r_E^2)[1 - \tfrac{3}{2}(x^2/r_E^2)] + (GM_M m/r_M^2)[1 - \tfrac{3}{2}(x^2/r_M^2)]$$

$$= -GM_E m/r_E^2 + GM_M m/r_M^2 - \text{ term in } x^2 \approx 0, \quad x^2 << r^2.$$

When we add the components perpendicular to the line joining Earth and the Moon, we get

$$F_\perp = -F_E \sin\theta - F_M \sin\phi = -GM_E mx/(r_E^2 + x^2)^{3/2} + GM_M mx/(r_M^2 + x^2)^{3/2}.$$

Using the same approximation, we get

$$F_\perp = (-M_e Gmx/r_E^3)[1 - \tfrac{3}{2}(x^2/r_E^2)] - (GM_M x/r_M^3)[1 - \tfrac{3}{2}(x^2/r_M^2)]$$

$$= -Gm(M_E/r_E^3 + (M_M/r_M^3)x, \quad x^2 << r^2.$$

The net force has magnitude

$$F_{\text{net}} = \boxed{Gm(M_E/r_E^3 + M_M/r_M^3)x, \text{ with a direction toward the original equilibrium point.}}$$