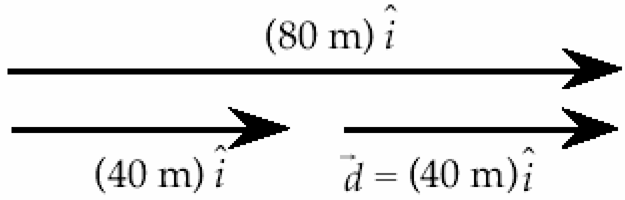
Chap. 2.

2. $(40\ \text{m})\hat{i}$.



4. We will choose a coordinate system with the origin at the start and x in the direction of the dash.

$0 \to 50$ m: $\vec{v}_{\text{av}} = \Delta\vec{x}/\Delta t = (50\ \text{m} - 0\ \text{m})\mathbf{i}/(5.61\ \text{s} - 0\ \text{s}) = \boxed{(8.9\ \text{m/s})\hat{i}}$.

$50 \to 100$ m: $\vec{v}_{\text{av}} = \Delta\vec{x}/\Delta t = (100\ \text{m} - 50\ \text{m})\mathbf{i}/(9.86\ \text{s} - 5.61\ \text{s}) = \boxed{(11.8\ \text{m/s})\hat{i}}$.

$0 \to 100$ m: $\vec{v}_{\text{av}} = \Delta\vec{x}/\Delta t = (100\ \text{m} - 0\ \text{m})\mathbf{i}/(9.86\ \text{s} - 0\ \text{s}) = \boxed{(10.1\ \text{m/s})\hat{i}}$.

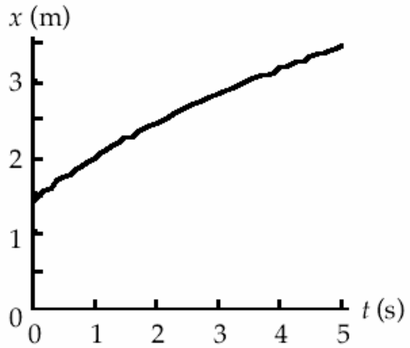
8. We take north as the y-axis. For the total trip

$$\vec{v}_{\text{av}} = \Delta\vec{y}/\Delta t = (30\ \text{mi} + 20\ \text{mi})\hat{j}/[(60\ \text{min} + 20\ \text{min})(1\ \text{h}/60\ \text{min})] = \boxed{(38\ \text{mi/h})\hat{j}}.$$

For the first half: $\vec{v}_{\text{av}} = \Delta\vec{y}/\Delta t = (30\ \text{mi})\hat{j}/[(40\ \text{min})(1\ \text{h}/60\ \text{min/h})] = \boxed{(45\ \text{mi/h})\hat{j}}$.

For the second half: $\vec{v}_{\text{av}} = \Delta\vec{y}/\Delta t = (20\ \text{mi})\hat{j}/[(40\ \text{min})(1\ \text{h}/60\ \text{min/h})] = \boxed{(30\ \text{mi/h})\hat{j}}$.

16. (a)



(b) For the average velocity, we have

$$\vec{v}_{\text{av}} = \frac{\Delta\vec{x}}{\Delta t} = \frac{\sqrt{(2.0\ \text{m/s}^2)(t_f + 1.0\ \text{s})} - \sqrt{(2.0\ \text{m/s}^2)(t_i + 1.0\ \text{s})}}{t_f - t_i}\hat{i}.$$

Between 1.0 s and 5.0 s, this gives

$$\vec{v}_{\text{av}} = \frac{\Delta\vec{x}}{\Delta t} = \frac{\sqrt{(2.0\ \text{m/s}^2)(5.0\ \text{s} + 1.0\ \text{s})} - \sqrt{(2.0\ \text{m/s}^2)(1.0\ \text{s} + 1.0\ \text{s})}}{5.0\ \text{s} - 1.0\ \text{s}}\hat{i} = (0.366\ \text{m/s})\hat{i}.$$

Between 2.0 s and 4.0 s, this gives

$$\vec{v}_{\text{av}} = \frac{\Delta\vec{x}}{\Delta t} = \frac{\sqrt{(2.0\ \text{m/s}^2)(4.0\ \text{s} + 1.0\ \text{s})} - \sqrt{(2.0\ \text{m/s}^2)(2.0\ \text{s} + 1.0\ \text{s})}}{4.0\ \text{s} - 2.0\ \text{s}}\hat{i} = (0.356\ \text{m/s})\hat{i}.$$

Between 2.8 s and 3.2 s, this gives

$$\vec{v}_{\text{av}} = \frac{\Delta\vec{x}}{\Delta t} = \frac{\sqrt{(2.0\ \text{m/s}^2)(3.2\ \text{s} + 1.0\ \text{s})} - \sqrt{(2.0\ \text{m/s}^2)(28\ \text{s} + 1.0\ \text{s})}}{3.2\ \text{s} - 2.8\ \text{s}}\hat{i} = (0.354\ \text{m/s})\hat{i}.$$

(c) For the instantaneous velocity, we have

$$\vec{v} = \frac{d\vec{x}}{dt} = \frac{1}{2}\frac{2.0\ \text{m/s}^2}{\sqrt{(2.0\ \text{m/s}^2)(t + 1.0\ \text{s})}}\hat{i}.$$

At 3.0 s this is

$$\vec{v} = \frac{d\vec{x}}{dt} = \frac{1}{2}\frac{2.0\ \text{m/s}^2}{\sqrt{(2.0\ \text{m/s}^2)(3.0\ \text{s} + 1.0\ \text{s})}}\hat{i} = (0.354\ \text{m/s})\hat{i}.$$

Note that the smaller the Δt for Δt centered at $t = 3.0$ s, the closer the average velocity approaches the instantaneous velocity.

21. We will take the origin as the location at $t = 0$ s, so we have $x = x_0 + v_0 t + \frac{1}{2}at^2 = \frac{1}{2}at^2$.

At $t = 8$ s: $x_8 = \frac{1}{2}a(8\ \text{s})^2$, at $t = 12$ s: $x_{12} = \frac{1}{2}a(12\ \text{s})^2$.

Then $x_{12} - x_8 = 64\ \text{m} = \frac{1}{2}a[(12\ \text{s})^2 - (8\ \text{s})^2]$, which gives

$a = \boxed{1.6\ \text{m/s}^2}$.

24. The average acceleration is found from $a_{av} = \Delta v/\Delta t$. We will take $\Delta t = (t + 0.5\ \text{s}) - (t - 0.5\ \text{s})$ and assume that for small Δt this is the acceleration at the time midpoint.

t, s	v, m/s	t, s	a_{av}, m/s²
0.0	0.0		
0.5	0.75	1.0	1.0
1.5	1.75	2.0	7.0
2.5	8.75	3.0	13.0
3.5	21.75	4.0	18.0
4.5	39.75	5.0	23.0
5.5	62.75	6.0	28.0
6.5	90.75	7.0	32.0
7.5	122.75		

26. From $x = At^2 + Be^{at}$ we can get the speed: $v = dx/dt = 2At + Bae^{at}$ and the acceleration: $a = dv/dt = 2A + Ba^2e^{at}$.

Putting the initial conditions into x, we get $-1.5\ \text{m} = A(0)^2 + Be^{a(0)}$, or $B = -1.5$ m.

Putting the initial conditions into v, we get $0.25\ \text{m/s} = 2A(0) + Bae^{a(0)}$;

$Ba = 0.25$ m/s and thus $a = -0.17\ \text{s}^{-1}$.

To obtain A, we use the second velocity condition: $0.045\ \text{m/s} = 2A(0.10\ \text{s}) + (-0.25\ \text{m/s})e^{-0.1/6,}$ which gives $A = -1.0\ \text{m/s}^2$.

At $t = 1.0$ s the acceleration is

$a = 2(-1.0\ \text{m/s}^2) + (-1.5\ \text{m/s})(-0.17\ \text{s}^{-1})^2e^{(-1/6)} = \boxed{-2.0\ \text{m/s}^2}$.

34. (a) For constant acceleration: $v = v_0 + at = 10\ \text{m/s} + (-0.2\ \text{m/s}^2)t$. Plug in $t = 1$ s and 2 s to obtain $v = \boxed{9.8\ \text{m/s}}$ and $\boxed{9.6\ \text{m/s}}$.

(b) At $t = 2$ s, $v = 9.6$ m/s. So $v_{av} = \frac{1}{2}(v_0 + v) = \frac{1}{2}(10\ \text{m/s} + 9.6\ \text{m/s}) = \boxed{9.8\ \text{m/s}}$.

38. Leg I: $\Delta x_1 = v_0 t + \frac{1}{2}a_1 t^2 = 0 + \frac{1}{2}(3.0\ \text{m/s}^2)(4\ \text{s})^2 = 24\ \text{m}$; $v_1 = v_0 + at = 0 + (3.0\ \text{m/s}^2)(4\ \text{s}) = 12\ \text{m/s}$.

Leg II: $\Delta x_2 = v_1 t + \frac{1}{2}a_2 t^2 = (12\ \text{m/s})(7\ \text{s}) + 0 = 84\ \text{m}$.

Leg III: $\Delta x_3 = v_1 t + \frac{1}{2}a_3 t^2 = (12\ \text{m/s})(15\ \text{s}) + \frac{1}{2}(1.0\ \text{m/s}^2)(15\ \text{s})^2 = 293\ \text{m}$;

$v_3 = v_1 + a_3 t = 12\ \text{m/s} + (1.0\ \text{m/s}^2)(15\ \text{s}) = 27\ \text{m/s}$.

Leg IV: $v_4^2 = v_3^2 + 2a(\Delta x_4)$; $0 = (27\ \text{m/s})^2 + 2(-2.5\ \text{m/s}^2)\,\Delta x_4$, which gives $\Delta x_4 = 146$ m.

Total displacement: $x = 24\ \text{m} + 84\ \text{m} + 293\ \text{m} + 146\ \text{m} = \boxed{547\ \text{m}}$.

42. We use a coordinate system with the origin at the corner and $t = 0$ when you turn the corner.

The bus position is given by

$$x_b = x_0 + v_0 t + \tfrac{1}{2}at^2 = 30\text{ m} + 0 + \tfrac{1}{2}(0.6\text{ m/s}^2)t^2$$

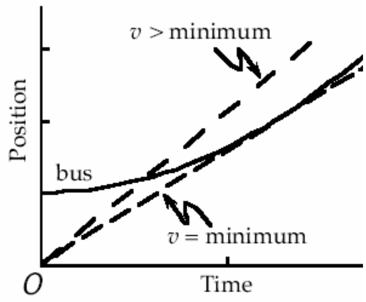
and its speed is

$$v_b = v_0 + at = 0 + (0.6\text{ m/s}^2)t.$$

Your speed is v_p and your position is given by

$$x_p = x_0 + v_p t = 0 + v_p t.$$

For you to catch the bus, your position must be the same as the bus's position. For you to catch the bus with a minimum speed, your speed must be the same as the bus's speed when you catch it. This can be seen from the x-t graph; the slope is the speed.



Thus, $x_b = x_p$, or

$$30\text{ m} + \tfrac{1}{2}(0.6\text{ m/s}^2)t^2 = v_p t$$

and $v_b = v_p$, or

$$(0.6\text{ m/s}^2)t = v_p.$$

These equations have two unknowns: t and v_p. Solving, we get $t = 10$ s and $v_p = \boxed{6.0\text{ m/s}}$.

50. We use a coordinate system with the origin at the initial position of B and up positive. The acceleration of A is negative, with the magnitude of the acceleration of B, which is positive. For the motion of the two objects, we have

$$y = y_0 + v_0 t + \tfrac{1}{2}at_1^{\,2};$$

$$y_A = (1.2\text{ m}) + 0 + \tfrac{1}{2}(-0.3\text{ m/s}^2)t^2,$$

$$y_B = 0 + 0 + \tfrac{1}{2}(+0.3\text{ m/s}^2)t^2.$$

The two objects bump into each other when $y_A = y_B$:

$$1.2\text{ m} + \tfrac{1}{2}(-0.3\text{ m/s}^2)t^2 = \tfrac{1}{2}(+0.3\text{ m/s}^2)t^2,\ \text{which gives}\ t = \boxed{2\text{ s}}.$$

58. We use a coordinate system with the origin at the release point and down positive. To find the time for the object to hit the ground, we have

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2;$$

$$10\text{ m} = 0 + 0 + \tfrac{1}{2}(+25.9\text{ m/s}^2)t^2,\ \text{which gives a positive answer of}\ t = \boxed{0.88\text{ s}}.$$

62. We use a coordinate system with the origin at the ground and up positive.
From the symmetry of the up and down motion, we know that the ball takes $\tfrac{1}{2}(3.6\text{ s}) = 1.8$ s to reach the highest point from the ground and $\tfrac{1}{2}(2.2\text{ s}) = 1.1$ s to reach the highest point after it passes the window. Thus, after being thrown, the ball takes 0.7 s to reach the window.
From $y = y_0 + v_{0y}t + \tfrac{1}{2}at^2$ we can write:

to reach the window: $10\text{ m} = 0 + v_0(0.7\text{ s}) + \tfrac{1}{2}(-g)(0.7\text{ s})^2$, and

to return to the ground: $0 = 0 + v_0(3.6\text{ s}) + \tfrac{1}{2}(-g)(3.6\text{ s})^2$.

Solving these two equations for the two unknowns gives us $v_0 = 18\text{ m/s}$ and $g = \boxed{9.85\text{ m/s}^2}$.

72. We use a coordinate system with the origin at the fingers with up positive. The point on the ruler that will be grabbed is L from the fingers. Its motion can be represented by

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2;\quad 0 = L + 0 + \tfrac{1}{2}(-g)t^2,\ \text{which gives}$$

$$t = (2L/g)^{1/2} = [2(5\text{ in})(1\text{ ft}/12\text{ in})/(32\text{ ft/s}^2)]^{1/2} = \boxed{0.16\text{ s}}.$$

76. We use a coordinate system with the origin at June's initial position and the positive direction toward Bill. Then $a_J = 1.0\text{ m/s}^2$ and $a_B = -0.9\text{ m/s}^2$:

$$x_J = x_{0J} + v_0 t + \tfrac{1}{2}a_J t^2 = 0 + 0 + \tfrac{1}{2}(1.0\text{ m/s}^2)t^2.$$

$$x_B = x_{0B} + v_0 t + \tfrac{1}{2}a_B t^2 = 20\text{ m} + 0 + \tfrac{1}{2}(-0.9\text{ m/s}^2)t^2.$$

When they meet: $x_J = x_B$, or $0.50t^2 = 20 - 0.45t^2$, which gives

$t = 4.6$ s and $x_J = \boxed{11\text{ m}}$.

80. Our coordinate system has the origin at the archer, up positive and $t = 0$ when the balloon is dropped.
For the balloon:

$$y_B = y_{0B} + v_{0B}t + \tfrac{1}{2}at^2 = 200\text{ m} + 0 + \tfrac{1}{2}(-9.8\text{ m/s}^2)t^2.$$

For the arrow:

$$y_A = y_{0A} + v_{0A}t + \tfrac{1}{2}gt^2 = 0 + (40\text{ m/s})(t - 5\text{ s}) + \tfrac{1}{2}(-9.8\text{ m/s}^2)(t - 5\text{ s})^2.$$

The arrow intercepts the balloon when $y_B = y_A$:

$$(200\text{ m}) - \tfrac{1}{2}(9.8\text{ m/s}^2)t^2 = (40\text{ m/s})(t - 5) - \tfrac{1}{2}(9.8\text{ m/s}^2)(t - 5\text{ s})^2.$$

The time of intercept is $t = 5.87$ s, which means $y = 200\text{ m} - \tfrac{1}{2}(9.8\text{ m/s}^2)(5.87\text{ s})^2 = \boxed{31\text{ m}}$.

82. We use a coordinate system with the origin at the ground and up positive. For all the weights, $v_0 = 0$ and $a = -g$. Take $t = 0$ when the string is released, and the lowest weight is at the ground.
The time for the jth sinker to hit the drum is found from:

$$y = y_{0j} + v_0t_j + \tfrac{1}{2}(-g)t_j^2;\ 0 = y_{0j} + 0 + \tfrac{1}{2}(-g)t_j^2,\text{ which gives } t_j = (2y_{0j}/g)^{1/2},\text{ with } j = 0, 1, 2, \ldots, n.$$

If we call $t_1 = (2L_0/g)^{1/2}$ the time for the second lowest weight to hit the ground, for equal time intervals we have $t_j - t_{j-1} = t_1$, $j = 2, 3, \ldots, n$. This gives the sequence $t_2 = 2t_1$, $t_3 = 3t_1$, $t_4 = 4t_1$, ….
In terms of the initial positions:

$$(2y_{0j}/g)^{1/2} = j\,(2L_0/g)^{1/2}, \quad\text{or}\quad y_{0j} = j^2L_0.$$

Because each y_0 is the sum of the corresponding L's, we have

$$y_{02} = L_0 + L_1 = 4L_0,\quad L_0 + L_1 + L_2 = 9L_0,\quad L_0 + L_1 + L_2 + L_3 = 16L_0, \ldots.$$

Successively combining these, we get $L_1 = 3L_0$, $L_2 = 5L_0$, $L_3 = 7L_0$, …, or $\boxed{L_j = (2j + 1)L_0,\ j = 1, 2, \ldots, n}$.