6. Because the velocities have opposite directions, momentum conservation gives us

$p_m + p_M = 0 = MV - mv$, which gives $V/v = m/M$.

The force of attraction provides the centripetal acceleration for each mass:

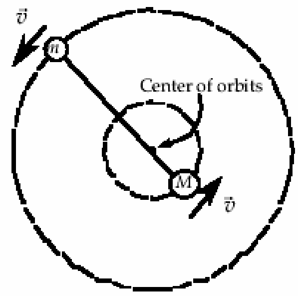
$F = mv^2/r = MV^2/R$. Thus

$R/r = MV^2/mv^2 = m/M$.

(a) For circular motion the angular speed is

$\omega_M = V/R = (mv/M)/(mr/M) = v/r = \omega_m$.

(b) From above, $\boxed{r/R = M/m}$.



16. (a) For this one-dimensional motion, we have $J = p_f - p_i$. For the satellite we get

$J_{\text{satellite}} = (850\ \text{kg})(0.45\ \text{m/s} - 0) = \boxed{3.8 \times 10^2\ \text{kg}\cdot\text{m/s}}$.

(b) The average force is

$F_{\text{av}} = J_{\text{satellite}}/\Delta t = (3.8 \times 10^2\ \text{kg}\cdot\text{m/s})/(0.85\ \text{s}) = \boxed{4.5 \times 10^2\ \text{N}}$.

20. (a) For the vertical motion, with down positive, we find Lois' speed at the ground from

$v^2 = v_0{}^2 + 2a\,\Delta y = 0 + 2(9.8\ \text{m/s}^2)(65\text{m})$, which gives $v = 36\ \text{m/s}$.

The impulse required to stop her is

$J = \Delta p = (52.5\ \text{kg})(0 - 36\ \text{m/s}) = \boxed{-1.9 \times 10^3\ \text{kg}\cdot\text{m/s, up}}$.

(b) If the force is constant, the acceleration is constant; so

$\Delta y' = \frac{1}{2}(v_0 + v)\,\Delta t$; $1.0\ \text{m} = \frac{1}{2}(36\text{m/s} + 0)\,\Delta t$, from which we get $\Delta t = \boxed{0.056\ \text{s}}$.

(c) We find the average force from

$F_{\text{av}} = J/\Delta t = (-1.87 \times 10^3\ \text{kg}\cdot\text{m/s})/(0.056\ \text{s}) = \boxed{-3.3 \times 10^4\ \text{N, up}}$.

For Lois, $mg = 5.1 \times 10^2$ N, so $F_{\text{av}} = \boxed{67\ mg}$. Ouch!

28. At the peak height the momentum is zero. If the first fragment moves straight down immediately after the explosion, the second fragment must move straight up. We use momentum conservation, with up positive:

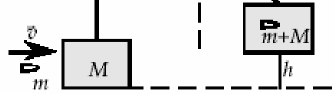
$P_i = P_{1f} + P_{2f}$

$0 = (1.1\ \text{kg})(-15\ \text{m/s}) + (2.7\ \text{kg})v_{2f}$, which gives $v_{2f} = \boxed{6.1\ \text{m/s, straight up}}$.

$\boxed{33}$. We let V be the speed of the block and bullet immediately after the collision and before the pendulum swings.

For this perfectly inelastic collision, we use momentum conservation:

$mv + 0 = (M + m)V$, which gives $V/v = m/(M + m)$.



(a) The fractional change in the kinetic energy is

$\Delta K/K_i = [(M + m)V^2 - mv^2]\,/\,mv^2$

$= [(M + m)/m](V/v)^2 - 1 = -M/(m + M)$,

so the fraction lost = $\boxed{M/(m + M)}$.

(b) For the pendulum motion we use energy conservation:

$\frac{1}{2}(M + m)V^2 = (m + M)gh$.

We combine this with the result from momentum conservation to get

$V = \sqrt{2gh} = mv/(m + M)$, which gives $v = [(m + M)/m]\sqrt{2gh}$.

39. When the large mass rebounds perfectly elastically from the wall, its kinetic energy does not change, so its velocity reverses direction with the same magnitude.

For the elastic collision of the two masses, we use momentum conservation:

$mv_1 + Mv_2 = mv_3 + Mv_4$;

$(0.126\text{ kg})(0.875\text{ m/s}) + (9.66\text{ kg})(-0.875\text{ m/s}) = (0.126\text{ kg})v_3 + (9.66\text{ kg})v_4$.

Because the collision is elastic, the relative speed does not change:

$v_1 - v_2 = -(v_3 - v_4)$, or

$0.875\text{ m/s} - (-0.875\text{ m/s}) = -(v_3 - v_4)$.

Combining these two equations, we get

$v_4 = -0.84\text{ m/s}$ and $v_3 = -2.59\text{ m/s}$, so the return speed is $\boxed{2.59\text{ m/s}}$.

42. (a) We use energy conservation to find the speed at the bottom:

$v = \sqrt{2gh} = \sqrt{2gL}$.

The speed of the lighter mass just before the collision is

$v_1 = [2(9.8\text{ m/s}^2)(0.95\text{m})]^{0.5} = \boxed{4.3\text{ m/s}}$.

For the elastic collision of the two masses, we use momentum conservation:

$mv_1 + Mv_2 = mv_3 + Mv_4$;

$(0.37\text{ kg})(4.3\text{ m/s}) + (0.56\text{ kg})(0) = (0.37\text{ kg})v_3 + (0.56\text{ kg})v_4$.

Because the collision is elastic, the relative speed does not change:

$v_1 - v_2 = -(v_3 - v_4)$, or $4.3\text{ m/s} - 0 = -(v_3 - v_4)$.

Combining these two equations, we get

$v_3 = \boxed{-0.9\text{ m/s}}$ and $v_4 = \boxed{3.4\text{ m/s}}$. The negative sign indicates a rebound.

(b) After the collision, we find the height to which the mass m rebounds from

$h_3 = v_3^2/2g = (-0.9\text{ m/s})^2/[2(9.8\text{ m/s}^2)] = \boxed{0.04\text{ m}}$.

51. We will take x to the right and y up. Because the speed of the bullet is so high, we ignore the effect of gravity and assume that it travels in a straight line until it hits the block. The time for the collision to take place is so small that we can ignore the effect of gravity on the system until after the collision.

For the perfectly inelastic collision of the two masses, we use momentum conservation:

x-direction: $m_{\text{block}}v_{\text{block},i} - m_{\text{bullet}}v_{\text{bullet},i}\cos 60° = (m_{\text{block}} + m_{\text{bullet}})v_x$;

$(0.80\text{ kg})(10\text{ m/s}) - (0.0050\text{ kg})(550\text{ m/s})\cos 60° = (0.805\text{ kg})v_x$, which gives $v_x = 8.2\text{ m/s}$.

y-direction: $0 - m_{\text{bullet}}v_{\text{bullet},i}\cos 60° = (m_{\text{block}} + m_{\text{bullet}})v_y$;

$0 + (0.0050\text{ kg})(550\text{ m/s})\sin 60° = (0.805\text{ kg})v_y$, which gives $v_y = 3.0\text{ m/s}$.

The velocity of the block immediately after the collision is

$\vec{v} = (8.2\text{ m/s})\hat{i} + (3.0\text{ m/s})\hat{j} = \boxed{8.7\text{ m/s, } 20° \text{ above the horizontal}}$.

52. For the collision of the two masses, we use momentum conservation:

$m_A\vec{v}_1 + m_B\vec{v}_2 = m_A\vec{v}_3 + m_B\vec{v}_4$;

$(0.15\text{ kg})[(-1.7\text{ m/s})\hat{i} - (2.0\text{ m/s})\hat{j}] + (0.22\text{ kg})(3.6\text{ m/s})\hat{i} = 0 + (0.22\text{ kg})\vec{v}_4$, which gives

$\vec{v}_4 = (2.4\text{ m/s})\hat{i} - (1.4\text{ m/s})\hat{j}$.

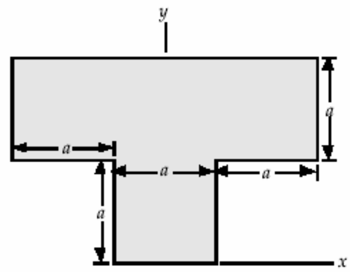
The kinetic energy of the second puck is

$K_4 = \frac{1}{2}m_Bv_4^2 = \frac{1}{2}(0.22\text{ kg})[(2.4\text{ m/s})^2 + (1.4\text{ m/s})^2] = \boxed{0.85\text{ J}}$.

64. The uniform area mass density of the object is

$\sigma = M/A = M/4a^2$.

We choose the x-axis along the bottom of the T and the y-axis up through the center of the T.



By symmetry, we have $\boxed{X = 0}$.

For the y-position of the center of mass, we treat the object as two rectangles and integrate:

$$Y = \frac{1}{M}\iint \sigma y \,dx\,dy = \frac{\sigma}{M}\left(\int_{-a/2}^{a/2} dx \int_0^a y\,dy + \int_{-3a/2}^{3a/2} dx \int_a^{2a} y\,dy\right)$$

$$= \frac{\sigma}{M}\left(a\,\frac{y^2}{2}\Big|_0^a + 3a\,\frac{y^2}{2}\Big|_a^{2a}\right) = \frac{Ma}{4a^2M}\left[\frac{a^2}{2}\Big|_0^a + 3\,\frac{(4a^2 - a^2)}{2}\Big|_a^{2a}\right] = \frac{5a}{4}.$$

69. We find the total mass of the stick from the integral:

$$M = \int_0^L \lambda(x)\,dx = \int_0^L \left(\frac{K}{x^2 + a^2}\right) dx = \frac{K}{a}\tan^{-1}\left(\frac{L}{a}\right).$$

We find the center of mass from the integral:

$$X = \frac{1}{M}\int_0^L \lambda(x)x\,dx = \frac{K}{M}\int_0^L \left(\frac{x}{x^2+a^2}\right) dx = \frac{K}{2M}\, ln\left(x^2+a^2\right)\Big|_0^L$$

$$= \quad a\,\frac{ln\left[(L^2+a^2)/a^2\right]}{2\tan^{-1}\left(L/a\right)}.$$

78. As in the previous problem, we need to find the horizontal velocity of the bullet (1) after it collides with the block (2). Use conservation of linear momentum: $m_1v_{1i} + m_2v_{2i} = m_1v_{1i} + m_2v_{2i}$. Here $v_{1i} = 450$ m/s and $v_{2i} = 0$. To find v_{1f}, first get v_{2f} from $v_{2f} = R_2/t = 0.55\text{ m}/0.45\text{ s} = 1.22$ m/s. (Note that the time of flight remains 0.45 s, as calculated in the previous problem, since h has not changed.) Thus

$v_{1f} = (m_1v_{1i} + m_2v_{2i} - m_2v_{2f})/m_1$

$= [(0.010\text{ kg})(450\text{ m/s}) + (0.40\text{ kg})(0) - (0.40\text{ kg})(1.22\text{ m/s})]/0.010\text{ kg} = 402\text{ m/s},$

and the range of the bullet after the collision is

$R_1 = v_{1f}\,t = (402\text{ m/s})(0.45\text{ s}) = \boxed{0.18\text{ km}}$.

84. We find the speed acquired by the sandbag during the collision by applying energy conservation to the swinging motion after collision:

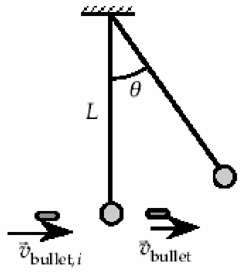
$\frac{1}{2}Mv_{\text{sandbag}}^2 = Mgh = MgL(1 - \cos\theta);$

$\frac{1}{2}v_{\text{sandbag}}^2 = (9.8\text{ m/s}^2)(1.2\text{ m})(1 - \cos 40°)$, which gives $v_{\text{sandbag}} = 2.4$ m/s.

For the conservation of momentum during the collision we write

$m_{\text{bullet}}v_{\text{bulleti}} + M(0) = m_{\text{bullet}}v_{\text{bullet}} + Mv_{\text{sandbag}};$

$(0.0080\text{ kg})(600\text{ m/s}) = (0.0080\text{ kg})(250\text{ m/s}) + M(2.4\text{ m/s})$, so $M = \boxed{1.2\text{ kg}}$.



85. We find the speed falling or rising through a height h from energy conservation: $\frac{1}{2}mv^2 = mgh$.

(a) The speed of each mass just before the collision is

$v_1 = \sqrt{2gh}$.

We choose the positive direction at the bottom in the direction of the motion of the larger mass. For the conservation of momentum during the collision we write

$Mv_1 + m(-v_1) = Mv_3 + mv_4$, with both final velocities assumed to be positive.

Because the collision is elastic, the relative speed does not change:

$v_1 - (-v_1) = -(v_3 - v_4)$ or $2v_1 = -v_3 + v_4$.

Combining these two equations, we get

$v_4 = [(3M - m)/(M + m)]v_1 = [(3M - m)/(M + m)]\sqrt{2gh}$.

We find the rebound height for the marble from

$v_4 = \sqrt{2gh'}$; $[(3M - m)/(M + m)]\sqrt{2gh} = \sqrt{2gh'}$, which gives $h' = [(3M - m)/(M + m)]^2 h$.

The overshoot is

$h' - h = \boxed{[8M(M - m)/(M + m)^2]h}$.

(b) For the conservation of momentum during the perfectly inelastic collision, we write

$Mv_1 + m(-v_1) = (M + m)v$, which gives $v = [(M - m)/(M + m)]\sqrt{2gh} = \sqrt{2gh''}$.

From this we get

$h'' = [(M - m)/(M + m)]^2 h$

and the "overshoot" is

$h'' - h = \boxed{-[4mM/(M + m)^2]h}$. (The combined masses will not reach the lip.)

89. We know from the symmetry that the center of mass lies on a line between the center of the styrofoam and the center of the solid. We choose the center of the styrofoam as origin and y along the line joining the centers. Then $X = 0$.

A uniform sphere has its center of mass at its center. We can treat the system as three spheres:

a sphere of density ρ and radius R with $Y_1 = 0$;

a sphere of density $-\rho$ and radius $\frac{1}{2}R$ with $Y_2 = \frac{1}{2}R$;

a sphere of density 5ρ and radius $\frac{1}{2}R$ with $Y_3 = \frac{1}{2}R$.

The last two are equivalent to a sphere of density 4ρ and radius $\frac{1}{2}R$ with $Y_4 = \frac{1}{2}R$.

We find the center of mass from

$Y = \Sigma m_i y_i / \Sigma m_i$

$= \{\rho(\frac{4}{3}\pi R^3)(0) + 4\rho[\frac{4}{3}\pi(\frac{1}{2}R)^3](\frac{1}{2}R)\} / \{\rho(\frac{4}{3}\pi R^3) + 4\rho[\frac{4}{3}\pi(\frac{1}{2}R)^3]\}$. This reduces to

$Y = \boxed{R/6 \text{ from the center of the styrofoam sphere}}$.

92. Rewrite $P(t) = P(t + \Delta t)$ as $P(t) = P(t + \Delta t) + mg\,\Delta t$. This adds an additional term to the right-hand-side of Equation (8-64), which now reads

$m\Delta v - (\Delta m)u_{ex} = mg\,\Delta t$.

Taking Δt to be infinitesimally small and reversing the sign in front of $\mathrm{d}m/\mathrm{d}t$, as in the textbook, we get the modified version of Equation (8-65):

$$-u_{ex}\frac{\mathrm{d}m}{\mathrm{d}t} = m\frac{\mathrm{d}v}{\mathrm{d}t} + mg.$$

Divide each term by m and move all the terms to the left-hand-side:

$$-\frac{u_{ex}}{m}\frac{\mathrm{d}m}{\mathrm{d}t} - \frac{\mathrm{d}v}{\mathrm{d}t} - g = -u_{ex}\frac{\mathrm{d}(\ln m)}{\mathrm{d}t} - \frac{\mathrm{d}v}{\mathrm{d}t} - g = \frac{\mathrm{d}}{\mathrm{d}t}\left(-u_{ex}\ln m - v - gt\right) = 0,$$

which means

$-u_{ex}\ln m - v - gt = \text{constant}$.

To find the constant, set $t = 0$, when $v = 0$ and $m = m_0$: $-u_{ex}\ln m_0 = \text{constant}$. Thus

$-u_{ex}\ln m - v - gt = -u_{ex}\ln m_0$, or

$v = u_{ex}(\ln m_0 - \ln m) - gt = u_{ex}\ln(m_0/m) - gt$. This is Equation (8-70).