10. The linear speed of the thread is the tangential speed of the surface of the spool:

$$v = \Delta x/\Delta t = (3\ \text{m})/(2\ \text{s}) = 1.5\ \text{m/s}.$$

The angular velocity of the spool is

$$\omega = v/r = (1.5\ \text{m/s})/(1.5 \times 10^{-2}\ \text{m}) = \boxed{100\ \text{rad/s along the axis}}.$$

16. As the wheel comes to rest

$\omega = w_1 t - w_2 t^2 = 0$, so $t = t_1 = w_1/w_2$. The angle turned during this time interval is

$$\theta = \int \omega\, \text{d}t = \int (w_1 t - w_2 t^2)\, \text{d}t = \tfrac{1}{2} w_1 t_1^2 - \tfrac{1}{3} w_2 t_1^3 = \tfrac{1}{2} w_1 (w_1/w_2)^2 - \tfrac{1}{3} w_2 (w_1/w_2)^3$$

$$= \boxed{w_1^3/6w_2^2}.$$

22. The rotational inertia of the dumbbell about the center is

$$I_1 = m(\tfrac{1}{2}L)^2 + m(\tfrac{1}{2}L)^2 = \tfrac{1}{2}mL^2.$$

The rotational kinetic energy is

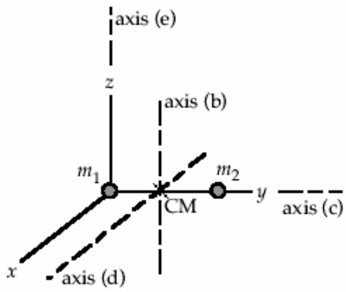
$$K_1 = \tfrac{1}{2} I_1 \omega^2 = \tfrac{1}{2}[\tfrac{1}{2}(1.5\ \text{kg})(0.62\ \text{m})^2][(36\ \text{rev/min})(2\pi\ \text{rad/rev})/(60\ \text{s/min})]^2 = \boxed{2.0\ \text{J}}.$$

The rotational inertia of the dumbbell about the end is

$$I_1 = 0 + mL^2 = mL^2.$$

The rotational kinetic energy is

$$K_1 = \tfrac{1}{2} I_1 \omega^2 = \tfrac{1}{2}[(1.5\ \text{kg})(0.62\ \text{m})^2][(36\text{rev/min})(2\pi\ \text{rad/rev})/(60\ \text{s/min})]^2 = \boxed{4.1\ \text{J}}.$$



28. (a) $\vec{R} = \sum m_i \vec{r}_i / \sum m_i$

$$= [m_1(0\ \text{m}, 0\ \text{m}, 0\ \text{m}) + m_2(0\ \text{m}, 1\ \text{m}, 0\ \text{m})]/(m_1 + m_2)$$

$$= \boxed{(0\ \text{m}, m_2/(m_1 + m_2)\ \text{m}, 0\ \text{m})}.$$

(b) $I_b = \sum m_i \vec{R}_i^{\,2} = m_1[m_2/(m_1 + m_2)]^2 + m_2\{1 - [m_2/(m_1 + m_2)]\}^2$

$$= \boxed{m_1 m_2/(m_1 + m_2)\ \text{kg}\cdot\text{m}^2}.$$

(c) $I_c = \sum m_i \vec{R}_i^{\,2} = m_1(0) + m_2(0)$

$$= \boxed{0}.$$

(d) $I_d = \sum m_i \vec{R}_i^{\,2} = m_1[m_2/(m_1 + m_2)]^2 + m_2\{1 - [m_2/(m_1 + m_2)]\}^2$

$$= \boxed{m_1 m_2/(m_1 + m_2)\ \text{kg}\cdot\text{m}^2}.$$

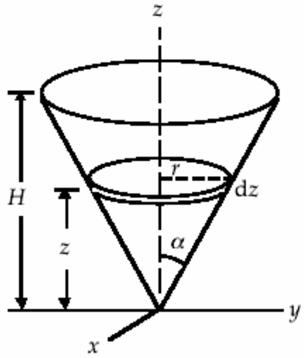
(e) $I_e = \sum m_i \vec{R}_i^{\,2} = m_1(0)^2 + m_2(1)^2$

$$= \boxed{m_2\ \text{kg}\cdot\text{m}^2}.$$

(f) The parallel-axis theorem for an axis through the origin and along the z-axis is

$$I_e = I_b + Md^2$$

$= m_1 m_2/(m_1 + m_2) + (m_1 + m_2)[m_2/(m_1 + m_2)]^2 = \boxed{m_2\ \text{kg}\cdot\text{m}^2}$, the same as part (e).



31. We choose a thin disk a distance z from the origin with thickness dz as the element, with mass d$m = \rho\pi r^2$ dz. The density is found from the mass of the cone: $M = \rho V = \rho(\tfrac{1}{3}\pi)\tan^2\alpha\, H^3$.
The rotational inertia of the disk about the z-axis is $\tfrac{1}{2}r^2\,\text{d}m$.
The integral for the rotational inertia of the cone is

$$I = \int_0^H \frac{1}{2} r^2 \rho\pi r^2\, \text{d}z.$$

From the diagram we have $r = z\tan\alpha$, so the integral becomes

$$I = \int_0^H \frac{1}{2}\rho\pi z^4 \left(\tan^4\alpha\right) \text{d}z. = \frac{1}{2}\rho\pi\frac{1}{5}z^5\left(\tan^4\alpha\right)\Big|_0^H$$

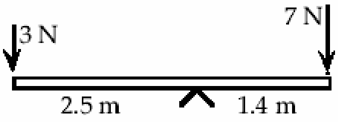
$$= \frac{1}{10}\rho\pi H^5\left(\tan^4\alpha\right) = \frac{1}{3}\rho\pi\left(\tan^2\alpha\right)H^3\frac{3}{10}\left(\tan^2\alpha\right)H^2;$$

$$I = \frac{3}{10}\left(\tan^2\alpha\right)MH^2.$$

43. (a) We choose the clockwise direction as positive.

$\tau_{pivot} = (7\text{ N})(1.4\text{ m}) - (3\text{ N})(2.5\text{ m}) = \boxed{2.3\text{ N}\cdot\text{m}}$.

(b) We see that $\boxed{\text{changing the 7-N force to 5.4 N}}$ will make the torque zero. An upward force of 8.4 N at the pivot will make the resultant force equal to zero.



48. The uniform distribution of the weight means that the normal force at each wheel is

$F_{NF} = F_{NB} = \frac{1}{2}mg = \frac{1}{2}(500\text{ kg})(9.8\text{ m/s}^2) = 2.45 \times 10^3\text{ N}.$

The maximum friction force at each wheel is

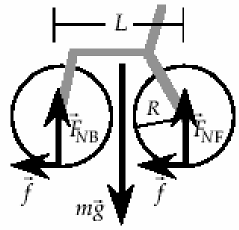
$f = \mu F_{NF} = 0.5(2.45 \times 10^3\text{ N}) = 1.23 \times 10^3\text{ N}.$

With the clockwise direction positive, the torque about the center of the front wheel is

$\tau = F_{NB}L + 2fR - \frac{1}{2}mgL$

$= (2.45 \times 10^3\text{ N})(1.5\text{ m}) + 2(1.23 \times 10^3\text{ N})(0.30\text{ m}) - \frac{1}{2}(500\text{ kg})(9.8\text{ m/s}^2)(1.5\text{ m})$

$= \boxed{7.4 \times 10^2\text{ N}\cdot\text{m}}$.



53. The spin angualar momentum of Earth is conserved:

$\frac{2}{5}MR_i^2\omega_i = \frac{2}{5}MR_f^2\omega_f$. Also, $T = 2\pi/\omega$. So

$T_f/T_i = \omega_i/\omega_f = (R_f/R_i)^2 = [(1 + 0.001\%)R_i/R_i]^2 \approx 1 + 2 \times 10^{-5}$,

and the length of a day would increase by

$\Delta T \approx (2 \times 10^{-5})T_i = (2 \times 10^{-5})(1\text{ day})(86400\text{ s/day}) = \boxed{2\text{ s}}$.

$\boxed{57}$. We write $\Sigma F_x = ma_x$ from the force diagram for the cylinder:

$Mg\sin\theta - f = Ma;$

We write $\Sigma\tau = I\alpha$ about the center of mass from the force diagram for the cylinder: $fR = I\alpha$.

For the rolling motion we have $a = R\alpha$.

By successively eliminating α and f, we get

$a = g\sin\theta/(1 + I/MR^2)$.

For the shell, $I = MR^2$; so $a = \frac{1}{2}g\sin\theta$.

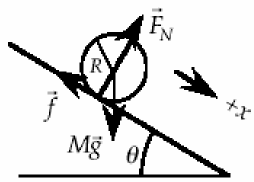
For the linear motion we use

$x = v_0t + \frac{1}{2}at^2 = 0 + \frac{1}{2}[\frac{1}{2}(9.8\text{ m/s}^2)\sin 20°](4\text{ s})^2 = \boxed{13.4\text{ m}}$.

For the solid cylinder, $I = \frac{1}{2}MR^2$; so $a = \frac{2}{3}g\sin\theta$.

For the linear motion we use

$x = v_0t + \frac{1}{2}at^2 = 0 + \frac{1}{2}[\frac{2}{3}(9.8\text{ m/s}^2)\sin 20°](4\text{ s})^2 = \boxed{17.9\text{ m}}$.



61. The rotational inertia of the system about its axis is

$I = \frac{1}{2}MR^2 + \frac{1}{2}mr^2$

$= \frac{1}{2}(0.80\text{ kg})(0.12\text{ m})^2 + \frac{1}{2}(0.10\text{ kg})(0.02\text{ m})^2 = 5.8 \times 10^{-3}\text{ kg}\cdot\text{m}^2.$

For the rolling motion, we write $\Sigma F_x = ma_x$ from the force diagram:

$(M + m)g\sin\theta - f = (M + m)a;$

We write $\Sigma\tau = I\alpha$ about the center of mass from the force diagram:

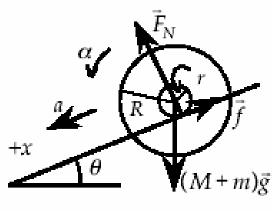
$fr = I\alpha$.

For the rolling motion without slipping, we have $a = r\alpha$.

By successively eliminating α and f, we get

$a = (M + m)gr^2\sin\theta/[I + (M + m)r^2]$

$= (0.90\text{ kg})(9.8\text{ m/s}^2)(0.02\text{ m})^2\sin 5°/[5.8 \times 10^{-3}\text{ kg}\cdot\text{m}^2 + (0.90\text{ kg})(0.02\text{ m})^2] = \boxed{0.050\text{ m/s}^2}$.



66. The initial rotational kinetic energy is

$K_i = \frac{1}{2}I\omega^2 = \frac{1}{2}(\frac{1}{2}MR^2)\omega^2.$

$= \frac{1}{4}(17\text{ kg})(0.33\text{ m})^2[(300\text{ rev/min})(2\pi\text{ rad/rev})/(60\text{ s/min})]^2 = \boxed{4.6 \times 10^2\text{ J}}.$

Because the rotational speed does not change, the increase in the kinetic energy is the kinetic energy of the 0.8-kg mass. The speed of this mass is $R\omega$, so its kinetic energy is

$K_m = \frac{1}{2}m(R\omega)^2 = \Delta K.$

The percentage change is

$\Delta K/K_i = 2m(R\omega)^2/MR^2\omega^2 = 2m/M = 2(0.8\text{ kg})/(17\text{ kg}) = \boxed{9\%}.$

69. Since the spool will not turn we have We write $\Sigma\tau = 0$. The force due to the weight of m_1 is $m_1 g$, and its lever arm about the central axis of the spool is R_1; so $\tau_1 = m_1 gR_1$. Similarly $\tau_1 = -m_1 gR_1$. Thus

$\Sigma\tau = \tau_1 + \tau_2 = m_1 gR_1 - m_2 gR_2 = 0$, which gives

$m_2 = \boxed{m_1R_1/R_2}.$

Since the spools does not rotate, its rotational inertia is irrelevant; so making the central cylinder hollow would $\boxed{\text{not}}$ change the result above.

71. (a) The angular speed of the cylinder is $\omega = v/R$. The angular momentum about the symmetry axis is

$L = I\omega = MR^2(v/R) = MRv = (3.0\text{ kg})(0.15\text{ m})(1.6\text{ m/s}) = \boxed{0.72\text{ kg}\cdot\text{m}^2/\text{s}}.$

(b) $K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}MR^2(v/R)^2 = \frac{1}{2}Mv^2 = \frac{1}{2}(3.0\text{ kg})(1.6\text{ m/s})^2 = \boxed{3.8\text{ J}}.$

(c) $K_{\text{total}} = K_{\text{lin}} + K_{\text{rot}} = \frac{1}{2}Mv^2 + \frac{1}{2}Mv^2 = 3.8\text{ J} + 3.8\text{ J} = \boxed{7.6\text{ J}}.$

73. The yo-yo will roll to the left with an angular velocity out of the page.

We write $\Sigma F_x = ma_x$ from the force diagram for the yo-yo:

$F - f_s = Ma;$

We choose out of the page as positive and write $\Sigma\tau = I\alpha$ about the center of mass from the force diagram for the yo-yo:

$f_s r - FR = I\alpha = \frac{1}{2}MR^2\alpha.$

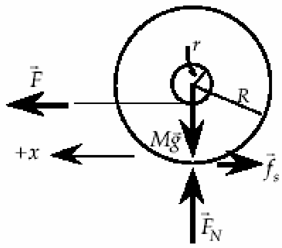
With rolling without slipping, we have $a = R\alpha$.

By successively eliminating α and a, we get

$F = 3f_s/(1 + 2r/R).$

Thus the force F will be maximum when the friction force is maximum, which is $f_{s\,\max} = \mu F_N = \mu Mg$. Therefore

$F_{\max} = 3\mu Mg/(1 + 2r/R) = \boxed{3\mu MgR/(R + 2r)}.$



74. (a) When the cylinder is in equilibrium, we write $\Sigma\tau = 0$ about the center of mass from the force diagram:

$T_0R = fR = 0$, which gives $T_0 = f$.

We write $\Sigma F_x = 0$ from the force diagram:

$f + T_0 - Mg\sin\theta = 0.$

When we combine the equations, we have $T_0 = \boxed{\frac{1}{2}Mg\sin\theta}.$

(b) We will assume the acceleration is up the plane.

For the rolling motion, we write $\Sigma F_x = ma_x$:

$f + T - Mg\sin\theta = Ma.$

We write $\Sigma\tau = I\alpha$ about the center of mass:

$TR - fR = I\alpha.$

For the rolling motion without slipping, we have $a = R\alpha$.

With $I = \frac{1}{2}MR^2$, by successively eliminating α and f, we get

$a = (2T - Mg\sin\theta)/(M + I/R^2) = \boxed{2(2T - Mg\sin\theta)/3M}.$

Note that this result holds if the acceleration is down the plane.

The tension and friction force will still be up the plane.