2002S Homework #05

Due 05/14/2002

1. The bottom of an infinite well located between $0 \le x \le b$ is changed to have the shape $V(x) = \varepsilon \sin \frac{\pi x}{b}$ within. Calculate the energy shifts for all the excited states to first order in ε.

2. The unperturbed Hamiltonian of a two-state system is represented by $H_0 = \begin{bmatrix} E_1^0 & 0 \\ 0 & E_2^0 \end{bmatrix}$.

   There is, in addition, a time-dependent perturbation $V(t) = \begin{bmatrix} 0 & \lambda \cos \nu t \\ \lambda \cos \nu t & 0 \end{bmatrix}$, ($\lambda$ real).

   a. At $t=0$ the system is known to be in the first state, represented by $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Using time-dependent perturbation theory and assuming E_1^0 - E_2^0 is not close to $\pm \hbar \nu$, derive an expression for the probability that the system can be found in the second state by $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ as a function of t ($t>0$).

   b. Why is this procedure not valid when E_1^0 - E_2^0 is close to $\pm \hbar \nu$?

3. Consider a composite system made up of two spin 1/2 objects. For $t<0$, the Hamiltonian does not dependent on spin and can be taken to be zero by suitable adjusting the energy scale. For $t>0$, the Hamiltonian is given by $H = \frac{4\Delta}{\hbar^2} \overrightarrow{S_1} \cdot \overrightarrow{S_2}$. Suppose the system is in $\left| + - \right\rangle$ for $t \le 0$. Find, as a function of time, the probability for being found in each of the following states $\left| + + \right\rangle$, $\left| + - \right\rangle$, $\left| - + \right\rangle$, and $\left| - - \right\rangle$:

   a. by solving the problem exactly.

   b. by solving the problem assuming the validity of first-order time-dependent perturbation theory with H as a perturbation switched on at $t=0$. Under what condition does (b) gives the correct results?