Homework #5

Due 11/08/2001

1. (a) Given the matrix representation of the *x*-component of the spin operator, S_x, $S_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, compute $\langle S_x \rangle_\psi$, which is the expectation value of S_x over a state $|\psi\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

   (b) Compute $\langle (\Delta S_x)^2 \rangle \equiv \langle S_x{}^2 \rangle - \langle S_x \rangle^2$.

   (c) Using the result in (b), check the **<u>generalized uncertainty relation</u>**,

$$\langle (\Delta A)^2 \rangle \langle (\Delta B)^2 \rangle \geq \frac{1}{4} |\langle [A, B] \rangle|^2,$$

   with $A \to S_x$, $B \to S_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$.

   (d) Do (a), (b), and (c) again for a new $|\psi\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

2. Show that the equality sign in the generalized uncertainty relation defined in 1(c) holds if the state $|\ \rangle$ in the relation satisfies $\Delta A |\ \rangle = \lambda \Delta B |\ \rangle$, with λ purely imaginary.