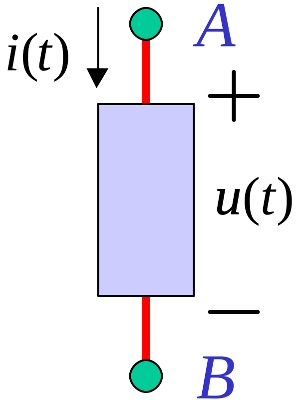
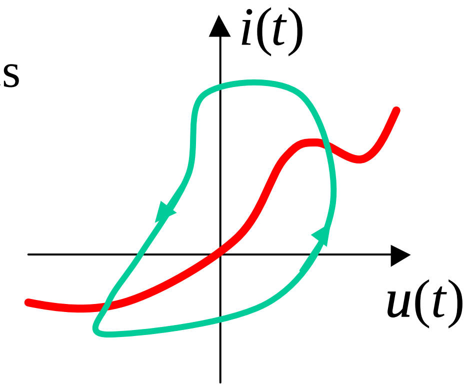
0.4 單埠元件與模型

單埠(one-port)

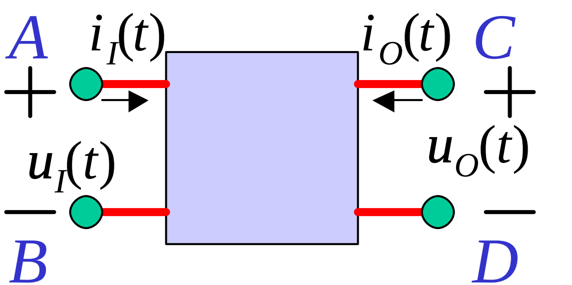


知道$i(t)= i[u(t)]$或$u(t) = u[i(t)]$即能瞭解此元件在電路中之行為及應用

Voltage-ampere characteristics
Voltage-ampere curve
Model
Constraint



雙埠(two-port)



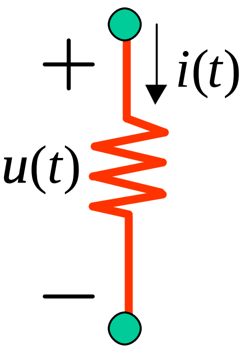
知道(i_I, u_I)與(i_O, u_O)之關係即能瞭解此元件在電路中之行為及應用

Two constraints

一些理想的單埠電路模型

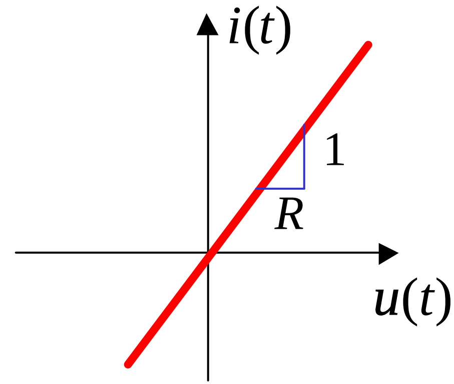
電阻

合乎Ohm's law的元件



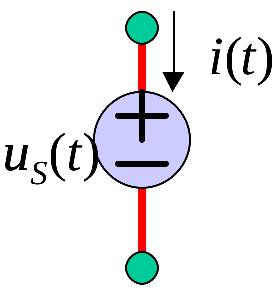
$$i(t)=\left(\frac{1}{R}\right)u(t)$$ Time domain

$$\mathbf{I}(w)=\left(\frac{1}{R}\right)\mathbf{V}(w)$$ Frequency domain



獨立電壓源

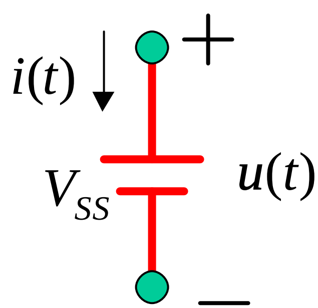
Independent voltage source



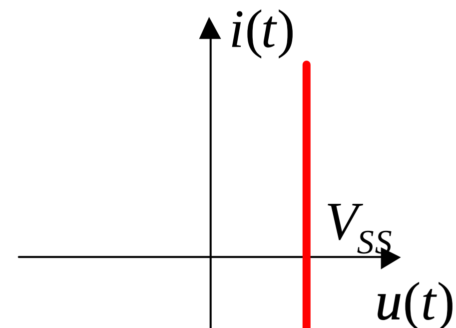
$$u(t)=u_S(t)$$
$$i(t)=?$$

(電流由外界電路決定)

直流獨立電壓源

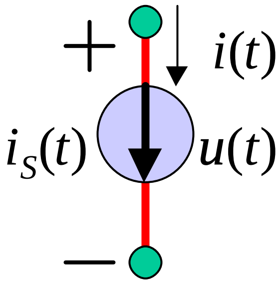


$$u(t)=V_{SS}$$
$$i(t)=?$$



獨立電流源

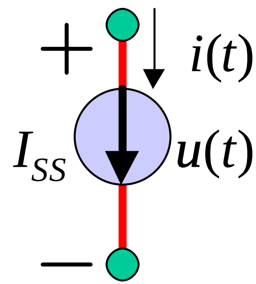
Independent current source



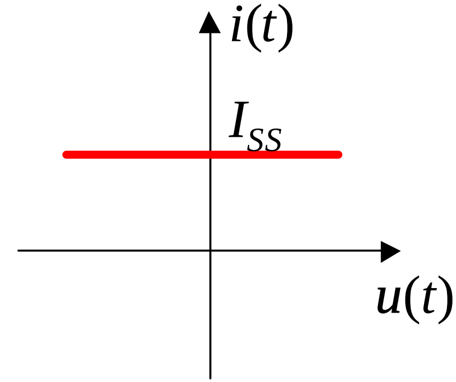
$$i(t) = i_S(t)$$
$$u(t) = ?$$

(電壓由外界電路決定)

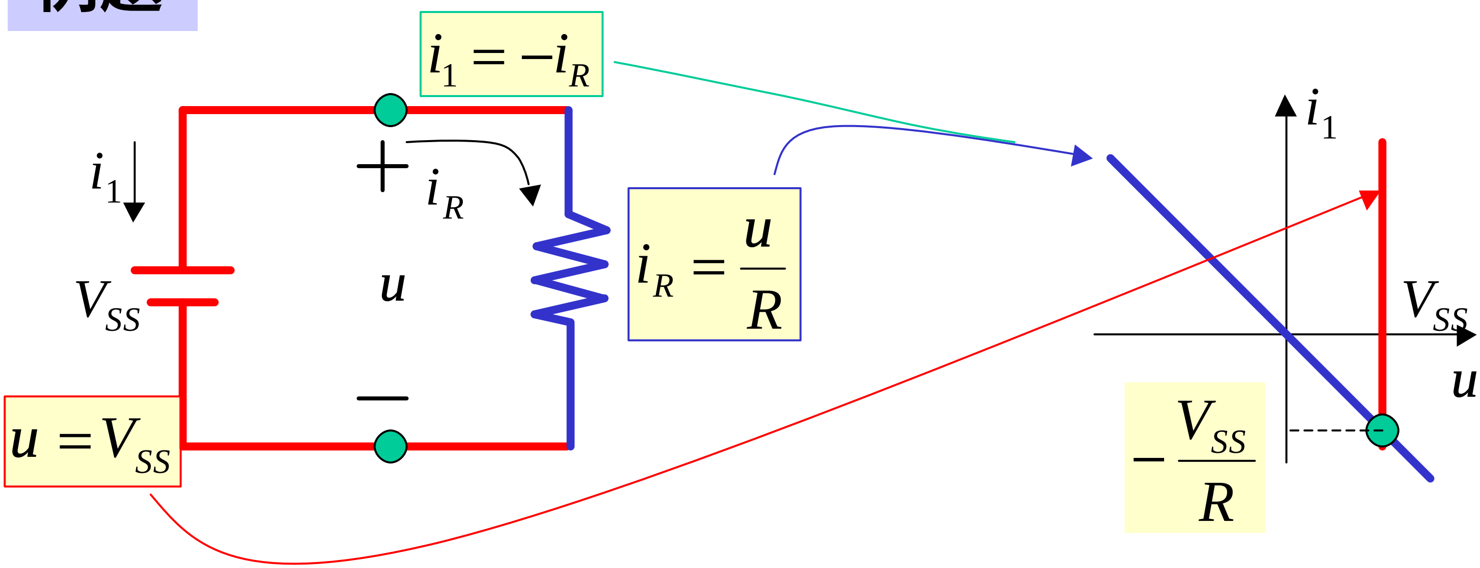
直流獨立電流源



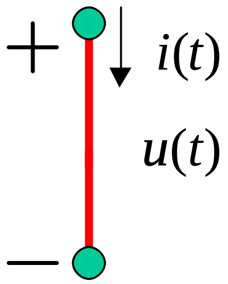
$$i(t) = I_{SS}$$
$$u(t) = ?$$



例題

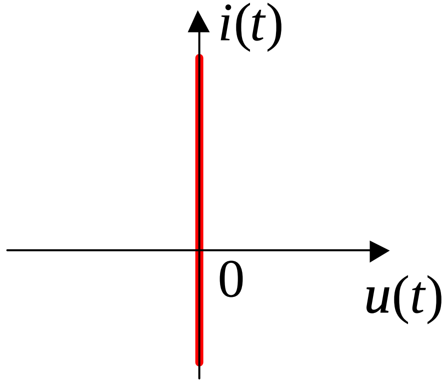


短路 Short circuit



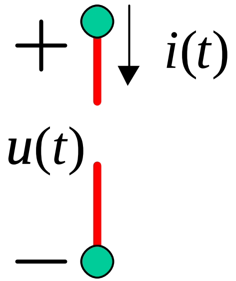
$$u(t)=0$$
$$i(t)=?$$

(電流由外界電路決定)



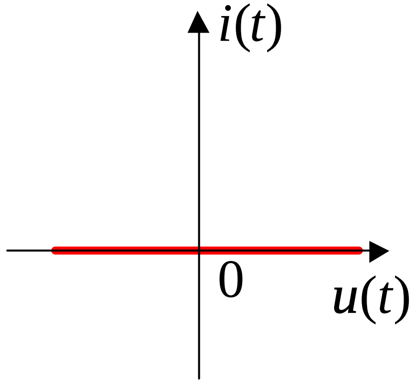
- 為電壓源之特例
- 當電壓源關掉時，即成為短路

開路 (斷路) Open circuit



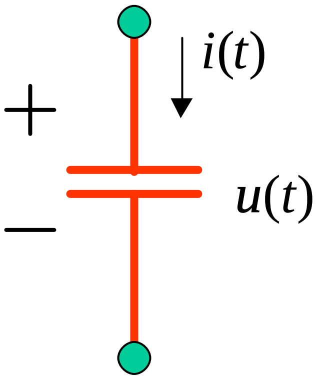
$$i(t)=0$$
$$u(t)=?$$

(電壓由外界電路決定)



- 為電流源之特例
- 當電流源關掉時，即成為開路

電容



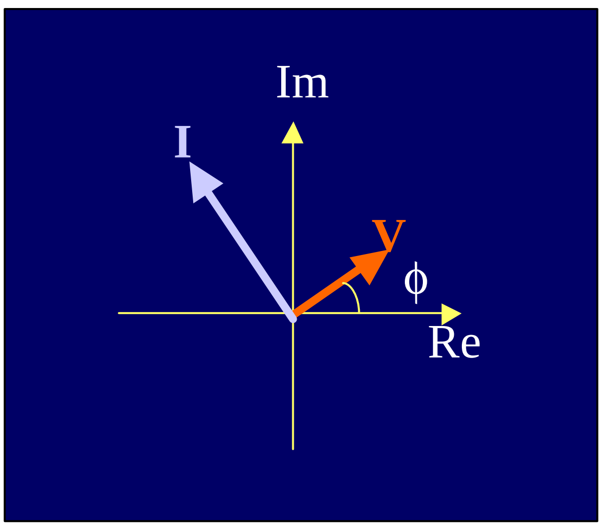
$$i(t) = C\frac{du(t)}{dt} \qquad u(t) = V_0\cos(wt + f) \rightarrow \mathbf{V} = V_0 e^{jf}$$

$$i(t) = -wCV_0\sin(wt + f) \rightarrow \mathbf{I} = jwCV_0 e^{jf} = jwC\mathbf{V}$$

$$i(t) = \mathrm{Re}\left[e^{jp} wCV_0 e^{jf} e^{jwt} e^{-\frac{jp}{2}}\right]$$

$$\mathbf{V} = \mathbf{I}\mathbf{Z}_C$$

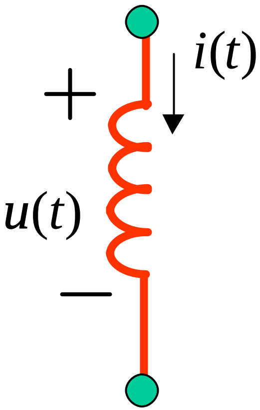
$$\mathbf{Z}_C = \frac{\mathbf{V}}{\mathbf{I}} = \frac{1}{jwC}$$



$$\mathbf{Z}_C = \frac{1}{jwC}$$ impedance

$$X_C \equiv \frac{1}{wC}$$ reactance

電感



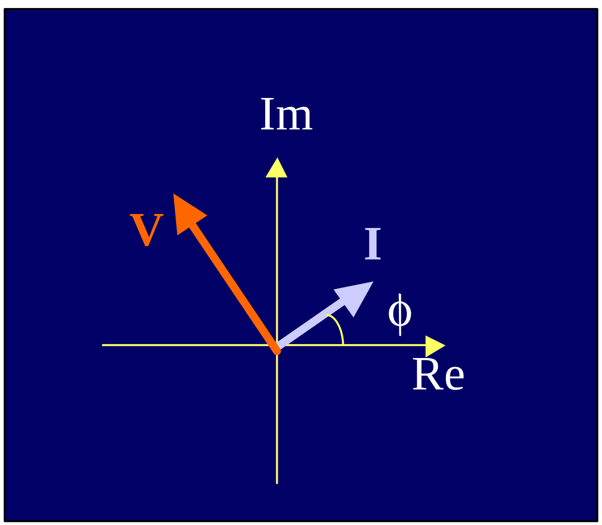
$$u(t) = L\frac{di(t)}{dt} \qquad i(t) = I_0\cos(wt + f) \rightarrow \mathbf{I} = I_0 e^{jf}$$

$$u(t) = -wLI_0\sin(wt + f) \rightarrow \mathbf{V} = jwLI_0 e^{jf} = jwL\mathbf{I}$$

$$u(t) = \mathrm{Re}\left[e^{jp} wLI_0 e^{jf} e^{jwt} e^{-\frac{jp}{2}}\right]$$

$$\mathbf{V} = \mathbf{I}\mathbf{Z}_L$$

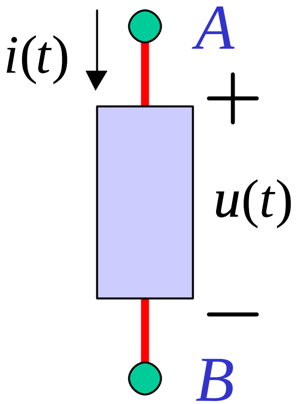
$$\mathbf{Z}_L = \frac{\mathbf{V}}{\mathbf{I}} = jwL$$



$$\mathbf{Z}_L = jwL$$

$$X_L = wL$$

單埠電路所消耗的功率



$$P(t) = i(t)u(t)$$

瞬時功率(instantaneous power)

$$P_{\text{av}}(t) = \lim_{t \to \infty} \frac{1}{t} \int_0^t i(t)u(t)dt$$

平均功率(average power)

考慮一AC電流通過某阻抗為Z的元件

$$\mathbf{Z} = |Z| e^{jf}$$

$$i(t) = I_{\max} \cos wt$$

$$u(t) = V_{\max} \cos(wt + f)$$

$$P(t) = i(t)u(t) = I_{\max} V_{\max} \cos wt \cos(wt + f)$$

$$P_{\text{av}}(t) = \lim_{t \to \infty} \frac{1}{t} \int_0^t i(t) u(t) dt$$

$$= \frac{1}{T} \int_0^T i(t) u(t) dt \quad \text{for periodic signal with period of } T$$

$$P_{\text{av}} = \frac{1}{T} \int_0^T I_{\max} V_{\max} \cos wt \cos(wt + f) dt$$

$$= \frac{I_{\max} V_{\max}}{T} \int_0^T \cos wt [\cos wt \cos f - \sin wt \sin f] dt$$

$$= \frac{I_{\max} V_{\max}}{T} \int_0^T [\cos^2 wt \cos f - \cos wt \sin wt \sin f] dt$$

$$= \frac{I_{\max} V_{\max}}{2} \cos f$$

$$V_{\max} = I_{\max} |Z|$$

$$P_{\text{av}} = \frac{I_{\max} V_{\max}}{2} \cos f = \frac{I_{\max}^2 |Z|}{2} \cos f = \frac{V_{\max}^2}{2|Z|} \cos f$$

我們稱cosφ**為功率因子**(power factor)**，**φ**恰為元件或電路阻抗的相角。**

對於純電容或電感的元件，ϕ為 $\pm\pi/2$，$\cos\phi=0$，平均消耗功率為零。電容與電感為能量儲存元件，不消耗能量。電阻則為能量消耗元件， $\cos\phi=1$。

對電阻而言

$$P_{\mathrm{av}R}=\frac{I_{\max}V_{\max}}{2}=\frac{I_{\max}^2R}{2}=\frac{V_{\max}^2}{2R}$$

我們可以定義一方便的平均值使得

$$I_{\mathrm{av}}^2=\frac{I_{\max}^2}{2}\qquad V_{\mathrm{av}}^2=\frac{V_{\max}^2}{2}$$

即

$$I_{\mathrm{av}}=\frac{I_{\max}}{\sqrt{2}}=0.707I_{\max}\qquad V_{\mathrm{av}}=\frac{V_{\max}}{\sqrt{2}}=0.707V_{\max}$$

這個平均恰好是所謂的方均根(rms)值

$$P_{\mathrm{av}}=I_{\mathrm{av}}V_{\mathrm{av}}\cos f=I_{\mathrm{av}}^2\,|Z|\cos f=\frac{V_{\mathrm{av}}^2}{|Z|}\cos f$$

$$=I_{\mathrm{rms}}V_{\mathrm{rms}}\cos f=I_{\mathrm{rms}}^2\,|Z|\cos f=\frac{V_{\mathrm{rms}}^2}{|Z|}\cos f$$

或用複數表示

$$P_{\mathrm{av}}=\mathrm{Re}[\mathbf{I}\mathbf{V}^*]=\mathrm{Re}[\mathbf{I}^*\mathbf{V}]$$

$$=I_{\mathrm{rms}}V_{\mathrm{rms}}\cos f$$

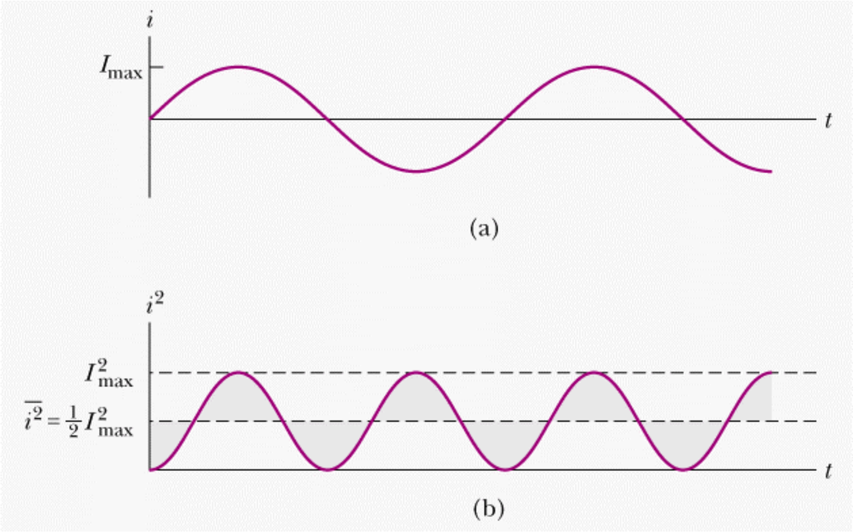
方均根值(root mean square, rms)

他的基本定義：

$$f_{\mathrm{rms}} = \sqrt{\frac{1}{T}\int_0^T [f(t)]^2 dt}$$

假如$f(t)$是平均為0的弦波函數，即 $f(t) = f_{\max}\cos(wt+q)$

$$f_{\mathrm{rms}} = \sqrt{\frac{1}{T}\int_0^T [f_{\max}\cos(wt+q)]^2 dt} = \frac{|f_{\max}|}{\sqrt{2}}$$



假如訊號不是弦波或平均值不是0，
則上式便不成立。