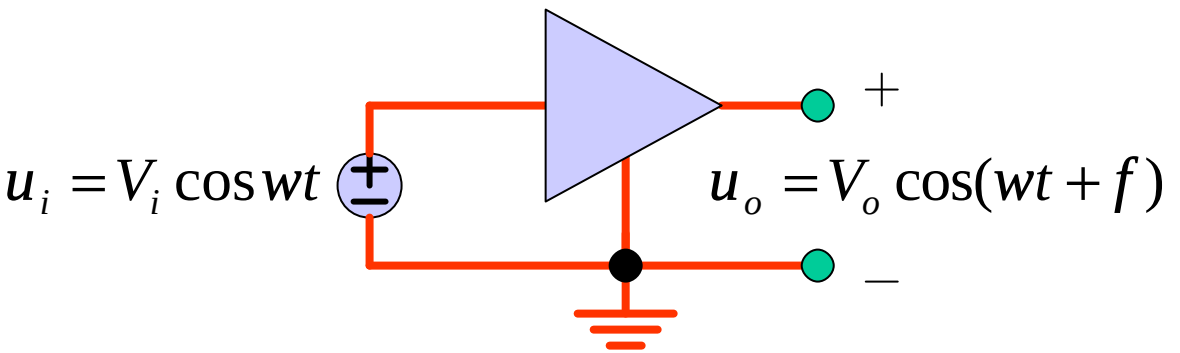
放大器的頻率響應 Frequency Response of an Amplifier

放大器的增益是頻率的函數，通常是用一個轉換函數(transfer function) $T(jw)$或$T(s)$來表示，是一個複數的函數。頻率響應來自電路中的電容或電感的成分。

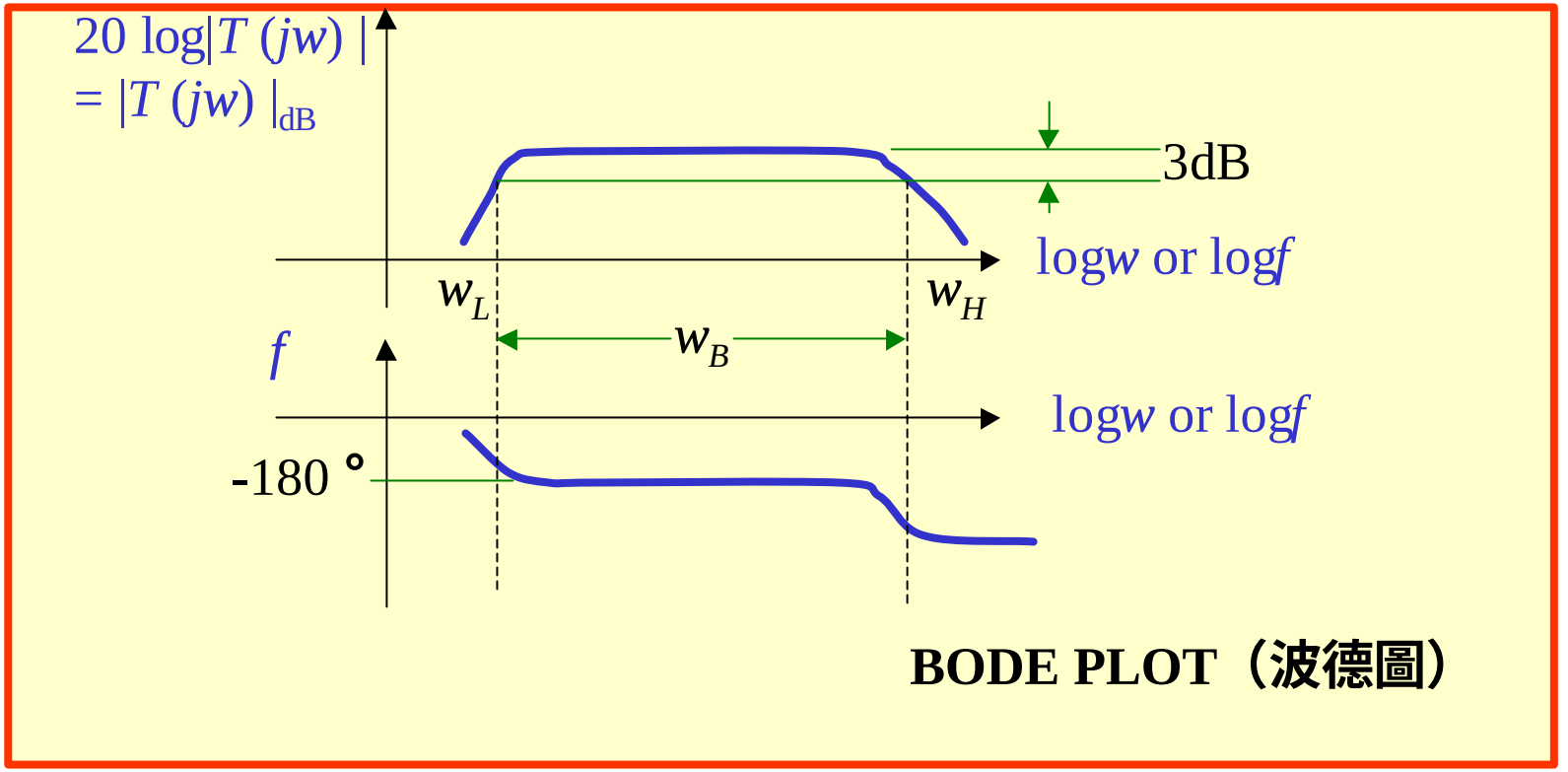
以一個電壓放大器，其轉換函數(transfer function) $T(jw)$或$T(s)$定義作

$$T(s)=T(jw)=\frac{\mathbf{V}_o(jw)}{\mathbf{V}_i(jw)} \qquad s=jw$$

$$|T(jw)|=\left|\frac{V_o(jw)}{V_i(jw)}\right|$$
$$\angle T(jw)=f$$



一個典型電壓放大器的頻率響應



$$u_i(t) = \sum V_i(w)\cos\left(wt + q(w)\right)$$

$$u_o(t) = \sum |T(jw)| V_i(w)\cos\left(wt + q(w) + f(w)\right)$$

Amplitude distortion

Phase distortion

dB（分貝）

相對的單位

電流或電壓增益

功率增益

給定一參考量(0dB)，其他量相對於參考量的大小

適合數量級範圍大的量的描述

$$A_u(\text{dB}) \equiv 20\log|A_u| = 20\log\left|\frac{V_o}{V_i}\right|$$

$$A_i(\text{dB}) \equiv 20\log|A_i| = 20\log\left|\frac{I_o}{I_i}\right|$$

$$A_p(\text{dB}) \equiv 10\log|A_p| = 10\log\left|\frac{P_o}{P_i}\right| \quad \text{(Power gain)}$$

$1\,\underset{A}{V} \rightarrow 2\,\underset{A}{V}$ $\quad 20\log 2 = +6\,\text{dB}$

$2\,\underset{A}{V} \rightarrow 1\,\underset{A}{V}$ $\quad 20\log\frac{1}{2} = -6\,\text{dB}$

$1\,\underset{A}{V} \rightarrow 10\,\underset{A}{V}$ $\quad 20\log 10 = +20\,\text{dB}$

$10\,\underset{A}{V} \rightarrow 1\,\underset{A}{V}$ $\quad 20\log\frac{1}{10} = -20\,\text{dB}$

$1\text{W} \rightarrow 2\text{W}$ $\quad 10\log 2 = +3\,\text{dB}$

$2\text{W} \rightarrow 1\text{W}$ $\quad 10\log\frac{1}{2} = -3\,\text{dB}$

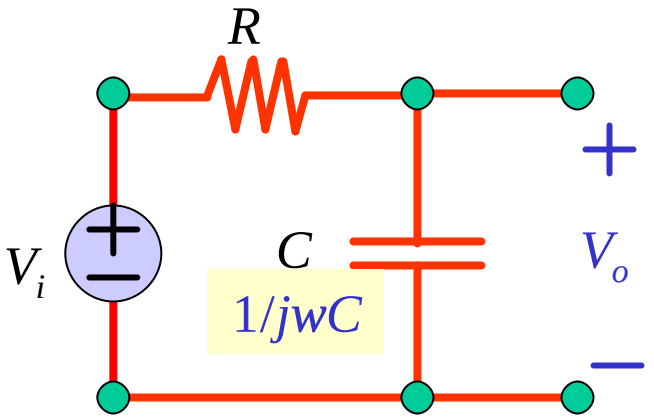
$1\text{W} \rightarrow 10\text{W}$ $\quad 10\log 10 = +10\,\text{dB}$

$10\text{W} \rightarrow 1\text{W}$ $\quad 10\log\frac{1}{10} = -10\,\text{dB}$

$$A_u = A_{u1}A_{u2}A_{u3} \qquad A_u(\text{dB}) = A_{u1}(\text{dB}) + A_{u2}(\text{dB}) + A_{u3}(\text{dB})$$

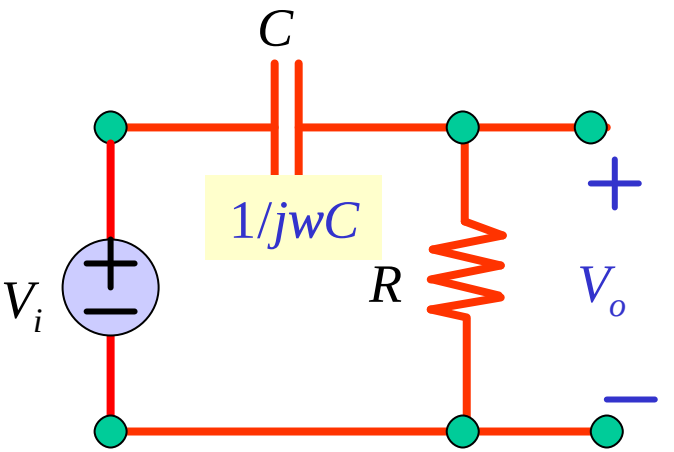
RC 電路

這裡討論的是最簡單的RC**電路，其中只包含一個有效電容，又稱做**STC (single time constant)**電路。**



Low pass network

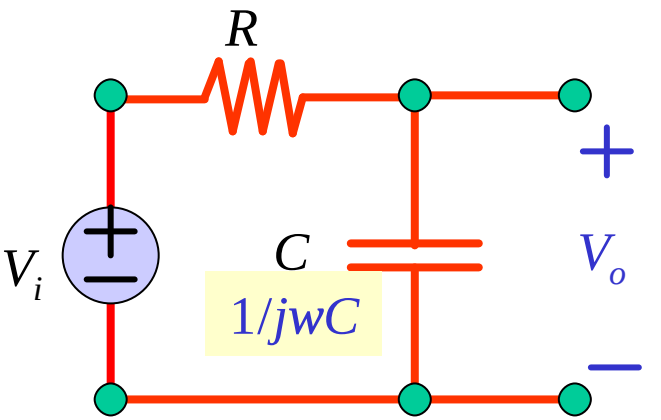
低通網路



High pass network

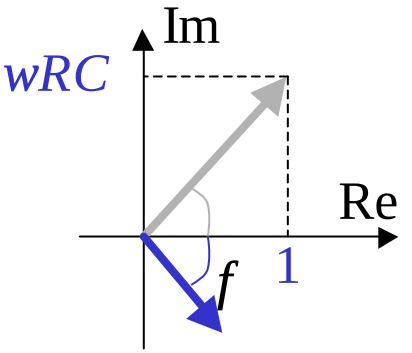
高通網路

Low pass network
低通網路

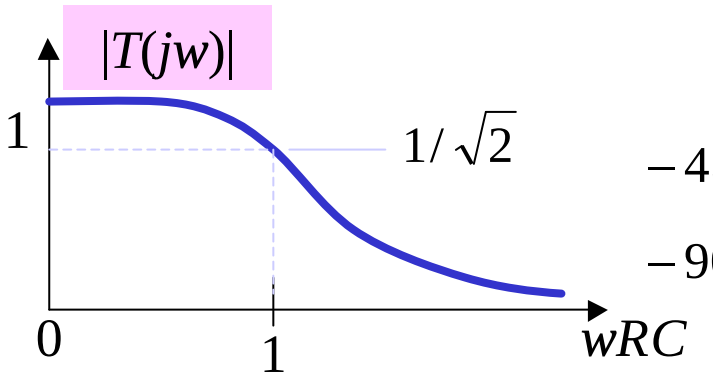


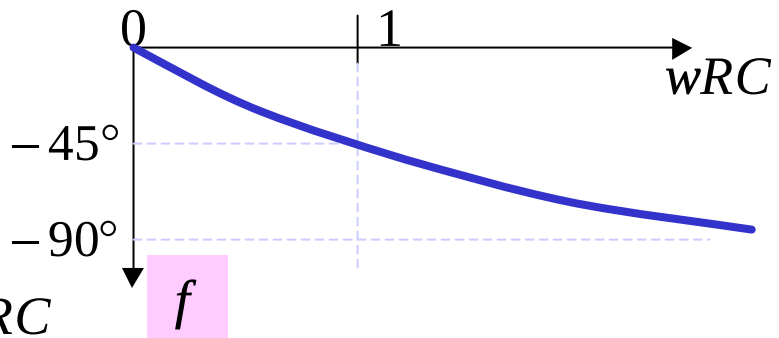
$$T(jw)=\frac{\mathbf{V}_o(jw)}{\mathbf{V}_i(jw)}=\frac{1/jwC}{R+1/jwC}=\frac{1}{1+jwRC}$$

$$|T(jw)|=\left|\frac{1}{1+jwRC}\right|=\frac{1}{(1+w^2R^2C^2)^{1/2}}$$



$$\angle T(jw)=\angle\left(\frac{1}{1+jwRC}\right)=-\angle(1+jwRC)=-\tan^{-1}(wRC)$$





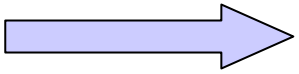
$$|T(jw)|(\text{dB}) = 20\log\left[\frac{1}{(1+w^2R^2C^2)^{1/2}}\right] = -10\log[1+w^2R^2C^2]$$

$$w^2R^2C^2 >> 1$$

$$|T(jw)|(\text{dB}) \approx -20\log wRC = -20\log 2pfRC = -20\log f - 20\log 2pRC$$

$$f \to 2f$$

$$|T(2f)|(\text{dB}) - |T(f)|(\text{dB}) = -20\log 2f + 20\log f = -20\log 2 = -6\text{dB}$$

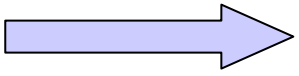


-6dB/Octave

STC**或一階濾波器**(first-order filter)**之特徵**

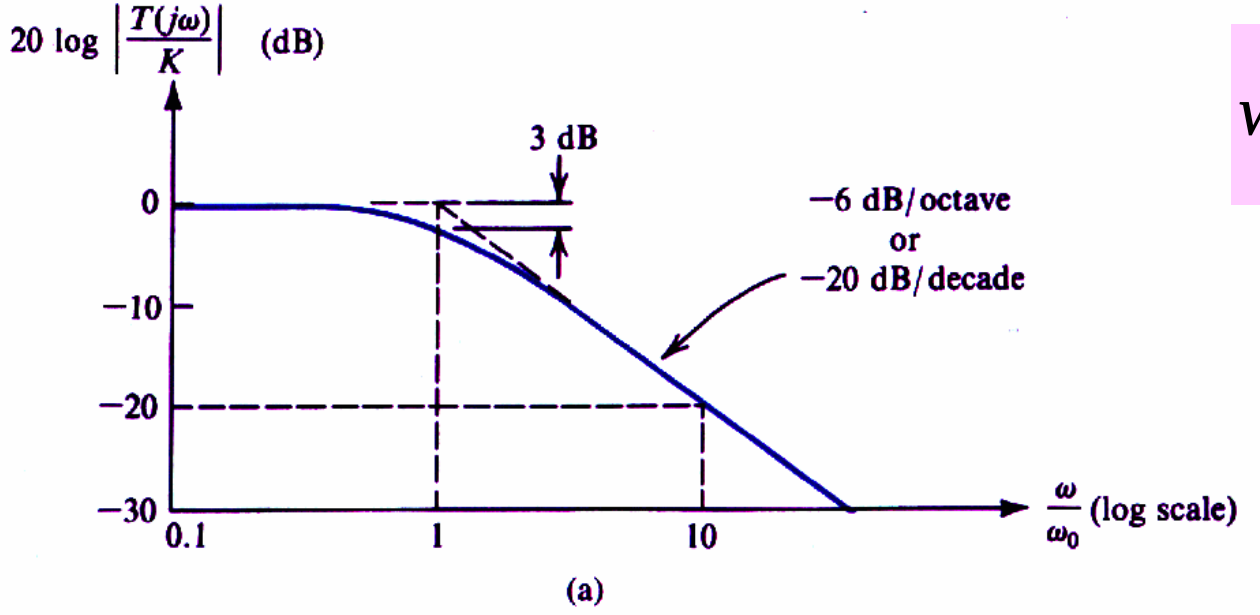
$$f \to 10f$$

$$|T(10f)|(\text{dB}) - |T(f)|(\text{dB}) = -20\log 10f + 20\log f = -20\log 10 = -20\text{dB}$$

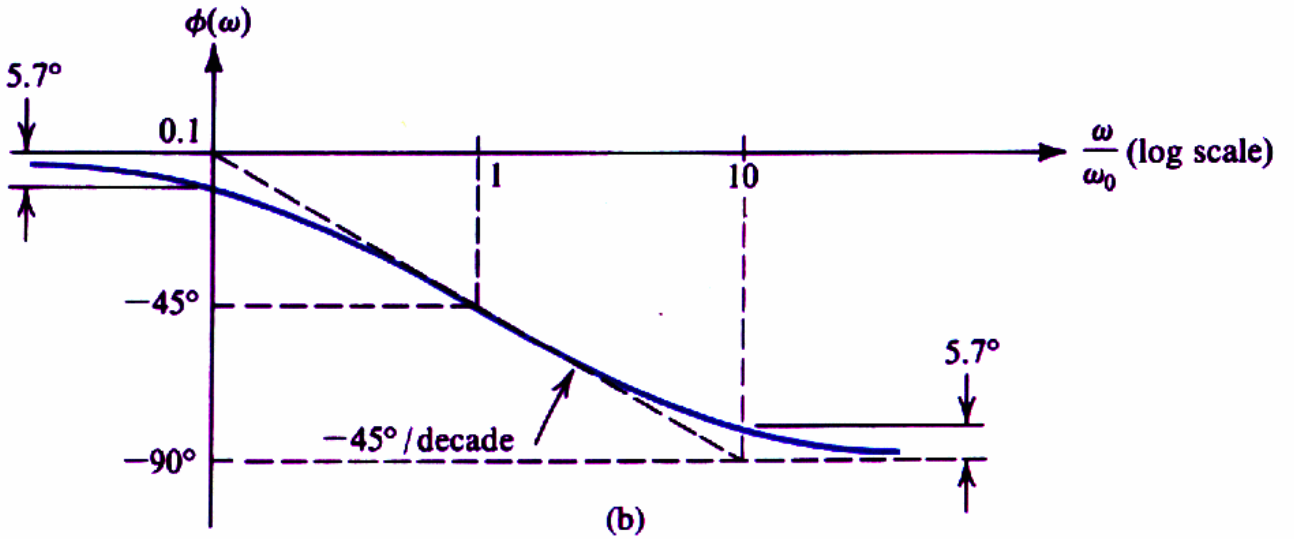


-20dB/Decade

低通電路的波德圖



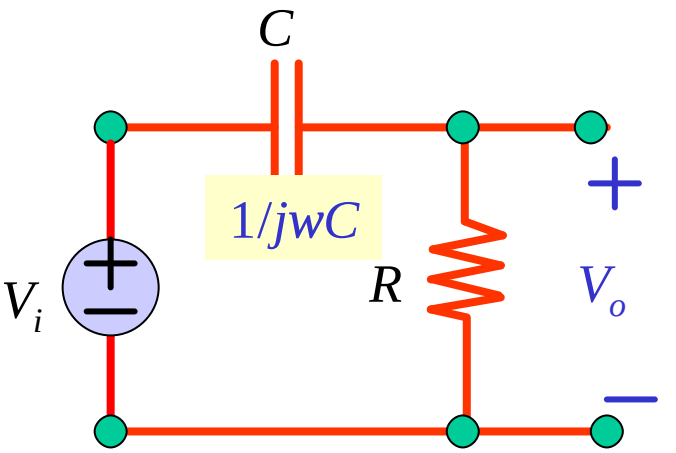
$$w_0 = \frac{1}{RC}$$



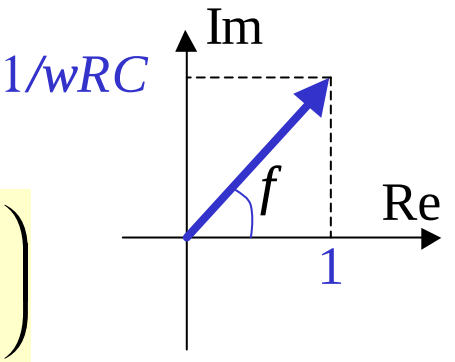
訊號衰減處有-90°相差。

High pass network
高通網路

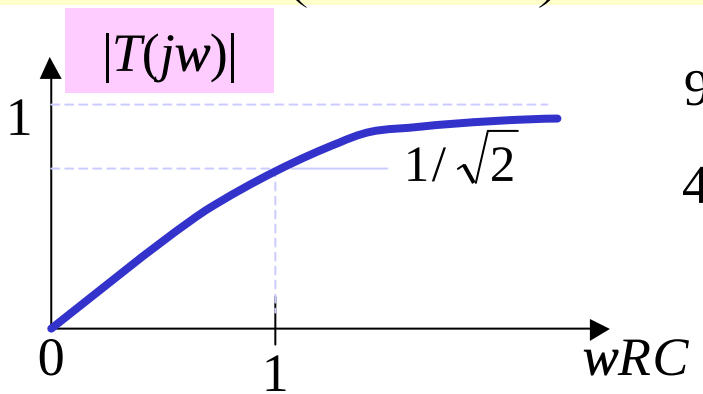
$$T(jw)=\frac{\mathbf{V}_o(jw)}{\mathbf{V}_i(jw)}=\frac{R}{R+\frac{1}{jwC}}=\frac{jwRC}{1+jwRC}$$

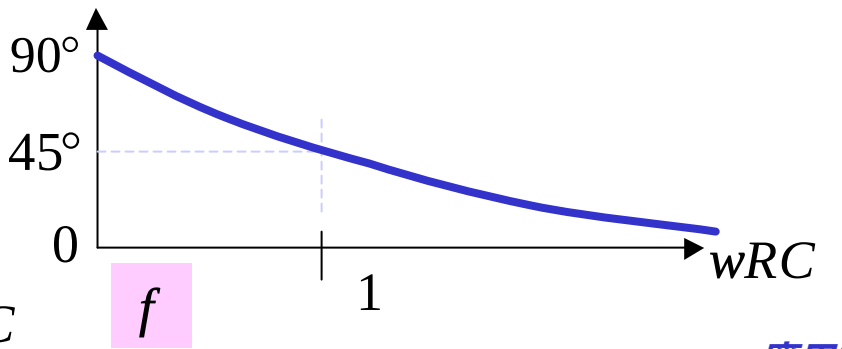


$$|T(jw)|=\left|\frac{jwRC}{1+jwRC}\right|=\frac{wRC}{(1+w^2R^2C^2)^{1/2}}$$

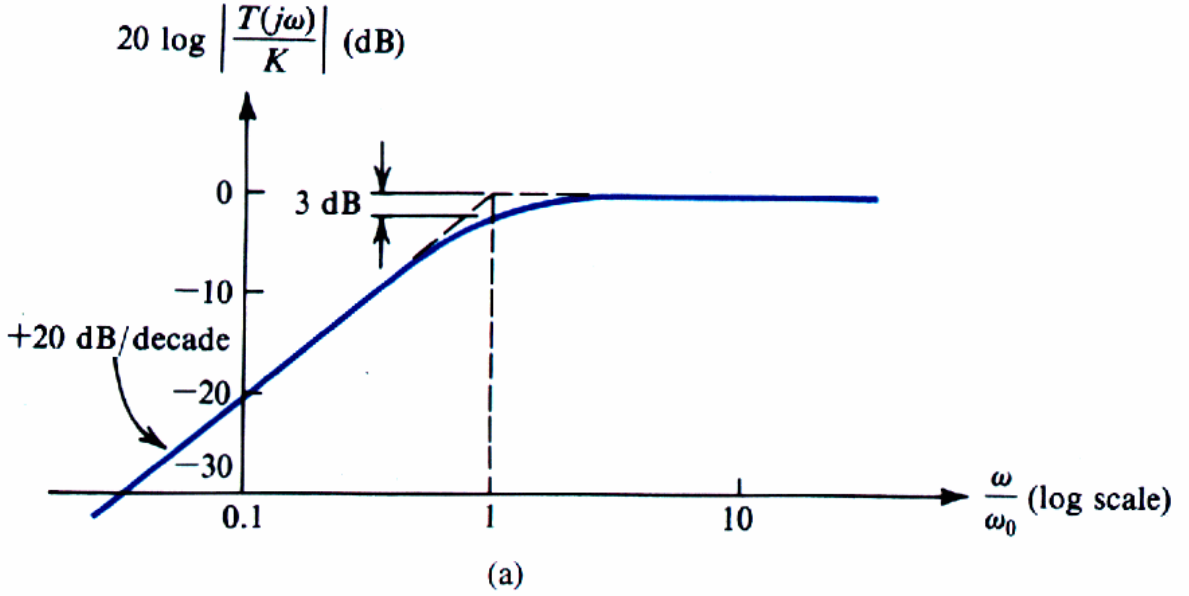


$$\angle T(jw)=\angle\left(\frac{jwRC}{1+jwRC}\right)=-\angle\left(1+\frac{1}{jwRC}\right)=\tan^{-1}\left(\frac{1}{wRC}\right)$$

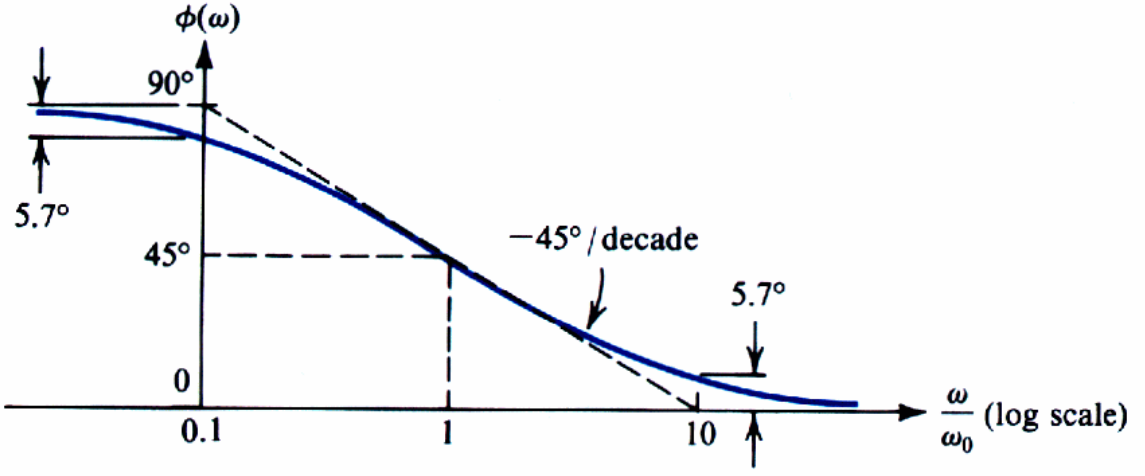




高通電路的波德圖



(a)



(b)

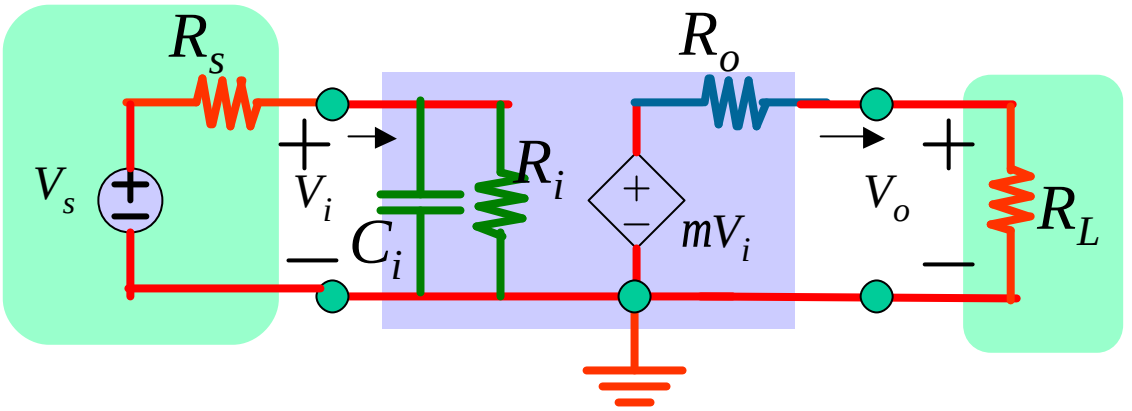
$$w_0 = \frac{1}{RC}$$

訊號衰減處有90°相差。

FREQUENCY RESPONSE OF STC NETWORKS

	Low-Pass (LP)	High-Pass (HP)
Transfer Function $T(s)$	$\dfrac{K}{1 + (s/\omega_0)}$	$\dfrac{Ks}{s + \omega_0}$
Transfer Function (for physical frequencies) $T(j\omega)$	$\dfrac{K}{1 + j(\omega/\omega_0)}$	$\dfrac{K}{1 - j(\omega_0/\omega)}$
Magnitude Response $\lvert T(j\omega)\rvert$	$\dfrac{\lvert K\rvert}{\sqrt{1 + (\omega/\omega_0)^2}}$	$\dfrac{\lvert K\rvert}{\sqrt{1 + (\omega_0/\omega)^2}}$
Phase Response $\angle T(j\omega)$	$-\tan^{-1}(\omega/\omega_0)$	$\tan^{-1}(\omega_0/\omega)$
Transmission at $\omega = 0$ (dc)	K	0
Transmission at $\omega = \infty$	0	K
3-dB Frequency	$\omega_0 = 1/\tau$; $\tau \equiv$ time constant $\tau = CR$ or L/R	

例題　輸入電容對電壓放大器的影響



(a)**計算轉換函數**$V_o/V_s(s)$**，並找出**dc gain**及**3dB frequency。

$$T(s)=\frac{V_o}{V_s}=\frac{V_i}{V_s}\cdot\frac{V_o}{V_i}=\frac{R_i\,//\,\dfrac{1}{sC_i}}{R_s+R_i\,//\,\dfrac{1}{sC_i}}\cdot m\cdot\frac{R_L}{R_o+R_L}$$

$$=\frac{R_i}{R_s+R_i+sC_iR_iR_s}\cdot m\cdot\frac{R_L}{R_o+R_L}$$

$$=\left[\frac{1}{1+sC_i(R_i\,//\,R_s)}\right]\left[\frac{R_i}{R_s+R_i}\,m\,\frac{R_L}{R_o+R_L}\right]$$

$$R_i\,//\,\frac{1}{sC_i}=\frac{R_i\dfrac{1}{sC_i}}{R_i+\dfrac{1}{sC_i}}$$

$$=\frac{R_i}{1+sCR_i}$$

dc gain
$s=0$

$$K=\frac{R_i}{R_s+R_i}m\frac{R_L}{R_o+R_L}$$

3dB frequency

$$w_0=\frac{1}{C_i(R_i\,//\,R_s)}=\left(C_i\frac{R_iR_s}{R_s+R_i}\right)^{-1}$$

(C_i×**看到外界的電阻**)$^{-1}$

(b)**若**R_s=20kΩ，R_i=100kΩ，C_i=60pF，m=144V/V，R_o=200Ω，R_L=1kΩ，**計算**dc gain，3dB frequency**及**unity gain frequency（gain**為**1**或**0dB**的頻率）。**

dc gain

$$K=\frac{100}{20+100}144\frac{20}{0.2+20}=100\rightarrow 40\text{dB}$$

3dB frequency

$$w_0=\frac{1}{C_i(R_i\,//\,R_s)}=\frac{1}{60\text{pF}\times(20\,//\,100)\text{k}\Omega}=10^6\text{ rad/s}$$

$$f_0=10^6/2p=159.2\text{kHz}$$

unity gain frequency

$$w_t=100w_0=10^8\text{ rad/s}$$

$$f_t=10^8/2p=15.92\text{MHz}$$

Fall-off rate –20dB/dec，gain**為**40dB，**故經**2dec**降為**0dB。

(c)**求下列輸入訊號對應之輸出訊號**$u_o(t)$:

$u_i(t)=0.1\sin 10^2 t,\text{V}$

w=10^2 rad/s=$w_o/10^4$，$|T|$~K=100，f~0°。**故** $u_o(t)=10\sin 10^2 t,\text{V}$

$u_i(t)=0.1\sin 10^5 t,\text{V}$

w=10^5 rad/s=$w_o/10$，$|T|$~K=100，f~-5.7°。**故** $u_o(t)=10\sin\left(10^5 t-5.7°\right),\text{V}$

更準確一點$|T|$=99.5，f=-$\tan^{-1}0.1$= -5.7° $u_o(t)=9.95\sin\left(10^5 t-5.7°\right),\text{V}$

$u_i(t)=0.1\sin 10^6 t,\text{V}$

w=10^6 rad/s=w_o，$|T|=100/\sqrt{2}=70.7, f=-45°$。**故**

$u_o(t)=7.07\sin\left(10^6 t-45°\right),\text{V}$

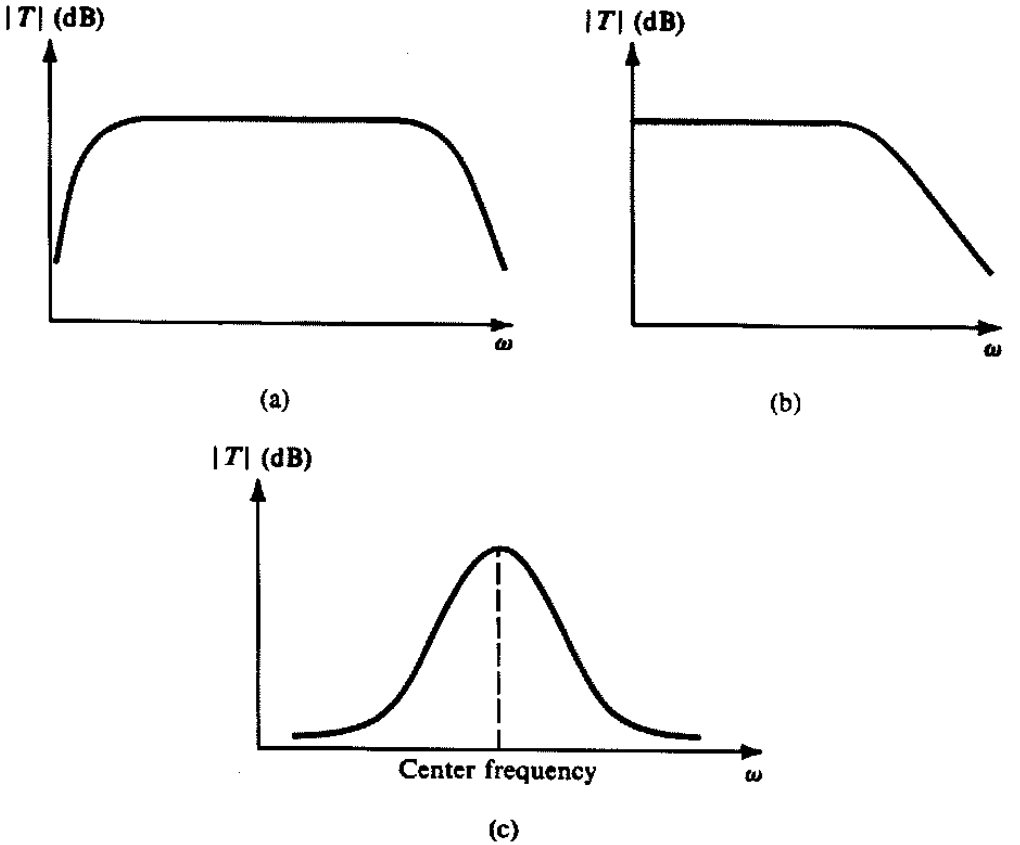
$u_i(t)=0.1\sin 10^8 t,\text{V}$

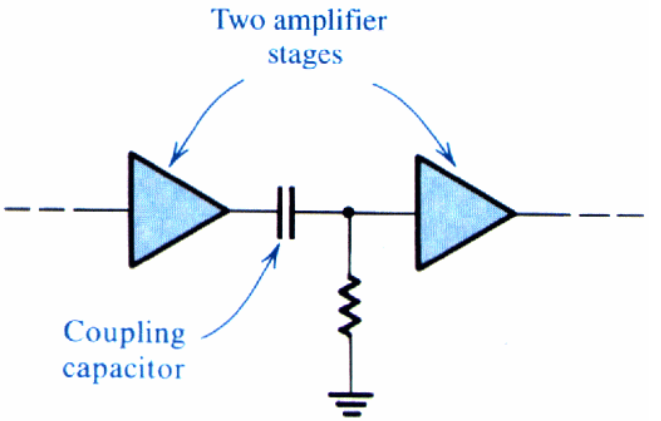
w=10^8 rad/s=100w_o，$|T|$~1，f~-90°。**故** $u_o(t)=0.1\sin\left(10^8 t-90°\right),\text{V}$

更準確一點$|T|$=1，f=-$\tan^{-1}100$= -89.4° $u_o(t)=0.1\sin\left(10^8 t-89.4°\right),\text{V}$

由頻率響應分類放大器

(a) a capacitively coupled amplifier, (b) a direct-coupled amplifier, (c) a tuned or bandpass amplifier.





(a)