Twisted impedance

Oliver Kent of Muirhead Aerospace Ltd recently commented on my formula for twisted-pair line impedance (Figure 1). “I am using coated wire with 1.3-mm outer diameter and 0.95-mm inner diameter,” Kent wrote. “The insulation is PTFE [polytetrafluoroethylene] with a dielectric constant, $\varepsilon_r$, of 2. Your formula contains the term $\ln((2S)/D)$. That term gives a negative value when twice the wire separation over the conductor diameter is less than 1. I appreciate any help or advice you could give regarding this matter.”

Kent may be interpreting the spacing as the distance between the wires. I don’t. In my system of units, I define the wire spacing from center to center (Figure 1). The spacing therefore always exceeds the diameter, so the ratio $2S/D$ always exceeds 2, and the logarithm cannot go negative.

The simple approximation in Figure 1 is not perfect. In the real world, if you could press the wires closer and closer together, the impedance would plummet to zero. The approximation doesn’t show that result. In the equation, when the metallic conductors touch with $S$ equal to $D$, the term $\ln((2S)/D)$ gives a minimum value of $\ln(2)=0.693$, which is incorrect.

I derived the approximation for use only in cases with a reasonably large spacing between wires, in which you may assume a uniform distribution of current around the periphery of each conductor. In a situation with closely spaced wires, the “proximity effect” generates a nonuniform distribution of current, with the greatest preponderance of current occurring on the inside-facing surfaces of the two conductors. Because the approximation does not take into account the proximity effect, it overestimates the impedance in situations with closely spaced wires.

For ordinary twisted-pair wires in a 100Ω configuration, the ratio $S/D$ is approximately 2. The redistribution of current due to the proximity effect in that case remains fairly modest, producing only a 15% increase in effective resistance and a negligible effect on impedance. In Kent’s case, the proximity effect will be more noticeable.

Another difficulty with the approximation involves the concept of “effective dielectric constant.” The fields surrounding the wires exist partly in the dielectric insulation and partly in the air surrounding the whole configuration. The effective dielectric constant, therefore, lies between that of air, which is 1, and that of PTFE, which is 2. The exact value takes into account the relative proportions of field energy in those two substances.

Unfortunately, unless you have a 2-D field solver handy, you can’t know in advance the relative proportions of field strength in air and insulation, so you can’t compute, from first principles, the effective dielectric constant. In that case, you should measure the velocity of propagation on a sample cable and use that measurement to determine the effective dielectric constant. The relation of velocity to dielectric constant is: velocity (m/sec)$=\sqrt{\varepsilon_r/\varepsilon_0}$.

Once you have obtained a value for the effective dielectric constant, plug that value into the approximation to determine the expected impedance and, more important, the expected change in impedance with changes in geometry. The effective dielectric constant does not vary much with small changes in the wire geometry.

Howard Johnson, PhD, of Signal Consulting, frequently conducts technical workshops for digital engineers at Oxford University and other sites worldwide. Visit his Web site at www.sigcon.com or e-mail him at howie03@sigcon.com.

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