Introduction to superconductivity
Textbook: “Introduction to superconductivity”
by A.C. Rose-Innes & E. H. Rhoderick 2nd Ed.

References:
“Introduction to superconductivity”
by M. Tinkham 2nd Ed.
“superconductivity” by C. P. Poole, Jr., H. A. Farach, and R. J. Creswick
Outline

1. Introduction-zero resistance
2. Perfect diamagnetism
3. Electrodynamics/The London theory
4. The critical magnetic field
5. Thermodynamics of the transition
6. The intermediate state
7. Transport currents in superconductors
8. The superconducting properties of small specimens
9. Ginzburg- Landau theory
10. The microscopic theory/BCS theory
11. Tunneling/Josephson effect
12. Type-II superconductivity
13. Hi-Tc superconductors
1. Introduction - Zero resistance
Zero resistance

- Perfectly pure metal
- Impure metal
- Residual resistivity $\rho_0$
- Critical temperature $T_C$
- Transition temperature
Discovery of superconductivity

H. Kamerlingh Onnes
(Leiden University)

He was the first to liquify helium (1908), for which he was awarded the Nobel prize in 1913, and he discovered superconductivity in 1911.

According to Onnes, "Mercury has passed into a new state, which on account of its extraordinary electrical properties may be called the superconductive state".

H. K. Onnes, Commun. Phys. Lab. 12, 120, (1911)
100 years in superconductivity

**Discovery of superconductivity**  H. Kamerlingh Onnes(1911) in Hg
1913 Nobel prize

**Perfect diamagnetism:** Meissner and Ochsenfeld(1933)

**London equation:** F. and H. London(1933)

**Ginzburg-Landau theory:** 1950s
2003 Nobel prize (with Abrikosov)

**Isotope effect:** H. Frohlich(1950)

**BCS theory:** J. Bardeen, L. Cooper and J.R. Schrieffer(1957) 1972 Nobel prize

**Tunneling:** Josephson (1957) 1973 Nobel prize

**Hi-Tc superconductivity:** J. G. Bednorz and K. A. Muller(1986) in Ba-La-Cu-O system. 1987 Nobel prize
Known superconductive elements

<table>
<thead>
<tr>
<th>Known Superconductive Elements</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Blue</strong> = at ambient pressure</td>
</tr>
<tr>
<td><strong>Green</strong> = only under high pressure</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Periodic Table</th>
<th>Lanthanide Series</th>
<th>Actinide Series</th>
</tr>
</thead>
<tbody>
<tr>
<td>58 Ce</td>
<td>59 Pr</td>
<td>90 Th</td>
</tr>
<tr>
<td>60 Nd</td>
<td>61 Pm</td>
<td>91 Pa</td>
</tr>
<tr>
<td>62 Sm</td>
<td>63 Eu</td>
<td>92 U</td>
</tr>
<tr>
<td>64 Gd</td>
<td>65 Tb</td>
<td>93 Np</td>
</tr>
<tr>
<td>66 Dy</td>
<td>67 Ho</td>
<td>94 Pu</td>
</tr>
<tr>
<td>68 Er</td>
<td>69 Tm</td>
<td>95 Am</td>
</tr>
<tr>
<td>70 Yb</td>
<td>71 Lu</td>
<td>96 Cm</td>
</tr>
<tr>
<td>100 Fm</td>
<td>101 Md</td>
<td>97 Bk</td>
</tr>
<tr>
<td>102 No</td>
<td>103 Lr</td>
<td>98 Cf</td>
</tr>
<tr>
<td>100 Fm</td>
<td>101 Md</td>
<td>99 Es</td>
</tr>
<tr>
<td>100 Fm</td>
<td>101 Md</td>
<td>99 Es</td>
</tr>
</tbody>
</table>
## Transition temperatures

<table>
<thead>
<tr>
<th>Element</th>
<th>Transition Temperature</th>
<th>Crystal Structure</th>
<th>Additional Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nb</td>
<td>9.25 K</td>
<td>BCC (Type 2)</td>
<td></td>
</tr>
<tr>
<td>Tc</td>
<td>7.80 K</td>
<td>HEX (Type 2)</td>
<td></td>
</tr>
<tr>
<td>Lead (Pb)</td>
<td>7.196 K</td>
<td>FCC</td>
<td></td>
</tr>
<tr>
<td>V</td>
<td>5.40 K</td>
<td>BCC (Type 2)</td>
<td></td>
</tr>
<tr>
<td>Lanthanum (La)</td>
<td>4.88 K</td>
<td>HEX</td>
<td></td>
</tr>
<tr>
<td>Tantalum (Ta)</td>
<td>4.47 K</td>
<td>BCC</td>
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<tr>
<td>Mercury (Hg)</td>
<td>4.15 K</td>
<td>RHL</td>
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<tr>
<td>Tin (Sn)</td>
<td>3.72 K</td>
<td>TET</td>
<td></td>
</tr>
<tr>
<td>Indium (In)</td>
<td>3.41 K</td>
<td>TET</td>
<td></td>
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<tr>
<td>Thallium (Tl)</td>
<td>2.38 K</td>
<td>HEX</td>
<td></td>
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<tr>
<td>Rhenium (Re)</td>
<td>1.697 K</td>
<td>HEX</td>
<td></td>
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<tr>
<td>Protactinium (Pa)</td>
<td>1.40 K</td>
<td>TET</td>
<td></td>
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<tr>
<td>Thorium (Th)</td>
<td>1.38 K</td>
<td>FCC</td>
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<tr>
<td>Aluminum (Al)</td>
<td>1.175 K</td>
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<tr>
<td>Gallium (Ga)</td>
<td>1.083 K</td>
<td>ORC</td>
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<td>Molybdenum (Mo)</td>
<td>0.915 K</td>
<td>BCC</td>
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<td>Zinc (Zn)</td>
<td>0.85 K</td>
<td>HEX</td>
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<td>Osmium (Os)</td>
<td>0.66 K</td>
<td>HEX</td>
<td></td>
</tr>
<tr>
<td>Zirconium (Zr)</td>
<td>0.61 K</td>
<td>HEX</td>
<td></td>
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<tr>
<td>Americium (Am)</td>
<td>0.60 K</td>
<td>HEX</td>
<td></td>
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<tr>
<td>Cadmium (Cd)</td>
<td>0.517 K</td>
<td>HEX</td>
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</tr>
<tr>
<td>Ruthenium (Ru)</td>
<td>0.49 K</td>
<td>HEX</td>
<td></td>
</tr>
<tr>
<td>Titanium (Ti)</td>
<td>0.40 K</td>
<td>HEX</td>
<td></td>
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<tr>
<td>Uranium (U)</td>
<td>0.20 K</td>
<td>ORC</td>
<td></td>
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<tr>
<td>Hafnium (Hf)</td>
<td>0.128 K</td>
<td>HEX</td>
<td></td>
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<tr>
<td>Iridium (Ir)</td>
<td>0.1125 K</td>
<td>FCC</td>
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<tr>
<td>Beryllium (Be)</td>
<td>0.023 K</td>
<td>HEX</td>
<td></td>
</tr>
<tr>
<td>Tungsten (W)</td>
<td>0.0154 K</td>
<td>BCC</td>
<td></td>
</tr>
<tr>
<td>Platinum (Pt)*</td>
<td>0.0019 K</td>
<td>FCC</td>
<td></td>
</tr>
<tr>
<td>Rhodium (Rh)</td>
<td>0.000325 K</td>
<td>FCC</td>
<td></td>
</tr>
</tbody>
</table>

*compacted powder

- MgB$_2$ 39K
- Nb$_3$Ge 23.2K
Transition temperature (Hi-Tc)

Bednorz and Muller, Z. Physik B64, 189, (1986)

Superconductivity at 93 K in a New Mixed-Phase Y-Ba-Cu-O Compound System at Ambient Pressure

M. K. Wu, J. R. Ashburn, and C. J. Torng
Department of Physics, University of Alabama, Huntsville, Alabama 35899

and

P. H. Hor, R. L. Meng, L. Gao, Z. J. Huang, Y. Q. Wang, and C. W. Chu
Department of Physics and Space Vacuum Epitaxy Center, University of Houston, Houston, Texas 77004
(Received 6 February 1987; Revised manuscript received 18 February 1987)

A stable and reproducible superconductivity transition between 80 and 93 K has been unambiguously observed both resistively and magnetically in a new Y-Ba-Cu-O compound system at ambient pressure. An estimated upper critical field $H_{c2}(0)$ between 80 and 180 T was obtained.

PACS numbers: 74.70.Ya
Persistent current

The inductance of the loop

\[ L = \mu_0 r \left[ \ln \frac{8r}{a} - 2 \right] \]

The resistance of the loop

\[ R = \frac{2r \rho}{a^2} \]

The time constant \( \tau = L/R \)

\[ i(t) = i(0)e^{-t/\tau} \]

\( \rho < 10^{-26} \ \Omega \text{ m} \)

For Cu \( \rho \sim 1.56 \ \mu\Omega \text{ cm} \)
Resist anceless circuit

The flux due to external field threading a closed loop \( \Phi_x = AB_{app} \)

The induction law yields:

\[
- A \frac{dB_a}{dt} = Ri + L \frac{di}{dt}
\]

For a resistanceless loop:

\[
- A \frac{dB_a}{dt} = L \frac{di}{dt}
\]

\[ \rightarrow Li + AB_a = \text{constant} \]

The total magnetic flux threading a closed loop \( \Phi = \Phi_x + \Phi_L = \text{constant} \)
Superconducting solenoid

The current is generated by the power supply P, and is adjusted by the rheostat R. Once the current is brought to the desired value, the switch XY can be closed. Since S and XY form a closed resistanceless loop, the magnetic field flux threaded by the loop remains the same. Now one can disconnect the power supply and the solenoid runs in the persistent mode.

Fig. 1.5. Superconducting solenoid.
2. Perfect diamagnetism
Perfect diamagnetism

**superconductor**

Perfect diamagnetism

Flux exclusion: zero field cooled

Flux expulsion: field cooled

\[ B = 0 \] Meissner effect

**Perfect conductor**

Constant magnetic flux

\[ \dot{B} = 0 \]

Recall the result deduced in page 14
Permeability

\[ B = \mu_0 (H + M) = \mu_0 (1 + \chi) H \]

\[ M = -H \]

\[ \chi = -1 \]

Magnetic susceptibility

Magnetic permeability \( \mu = 0 \)
$$B_{\text{app}} = \mu_0 H$$

$$M = -H \quad \text{(inside the sphere)}$$

A dipole field outside the sphere

$$B$$
Surface currents

\[ \nabla \times B = \mu_0 J \]

\[ J = \nabla \times M \]
Hole in Superconductor
Surface currents
Penetration depth

Boundary condition

\[ H'_{\|} = H''_{\|} \]
\[ H_{app} = H_{in} \]

For perfect diamagnetism

\[ B_{app} = \mu_0 H_{app} \]
\[ B_{in} = 0 \]

For a finite current density, the surface current is distributed within a depth, \( \lambda \)
London theory

\[ B(x) = B(0)e^{-x/\lambda} \]
Variation with temperature

\[ \frac{\lambda}{\lambda_0} = \frac{1}{(1-t^4)^{1/2}} \]

\[ t = \frac{T}{T_C} \]

The measurement of penetration depth

Self inductance of the solenoid, \( L \) is a function of penetration depth

The tank circuit form an oscillator of a typical frequency