6. We find the force produced by the string by differentiating the potential energy:
\[ F = -\frac{dU}{dx} = -\frac{d}{dx}(\frac{1}{2}m(x^2 + x^2))/dx = -(2x + 3\alpha^2). \]
The force exerted by the archer is the reaction to this force:
\[ F_{\text{archer}} = 2x + 3\alpha^2, \]
\[ F_{\text{string}} = -2x - 3\alpha^2. \]

10. We take \( y = 0 \) at the ground.
   (a) \( U = mgy = (5 \text{ kg})(9.8 \text{ m/s}^2)y = 49y. \)
   (b) \( U = mgy - 10 \text{ m} = 49y - 490. \)
   (c) \( U = mgy - 4 \text{ m} = 49y - 200. \)

20. (a) We choose \( U = 0 \) at \( x = 0 \). From the relation between force and potential energy, \( F = -\frac{dU}{dx} \), we see that a constant force means a constant slope on the potential energy plot. For this force the slope is \(-2.5 \text{ N} \) for negative \( x \) and \(+0.8 \text{ N} \) for positive \( x \).
   (b) At \( x = -0.5 \text{ m} \), the energy is only potential, with a value of \( 1.3 \text{ J} \). We show a line at constant energy of \( 1.3 \text{ J} \), which intercepts the plot at \( x = 1.6 \text{ m} \). Thus the particle travels \( 1.6 \text{ m} \) before stopping momentarily.

24. (a) For the energy \( E_1 \), motion is restricted to
   \( A < x < B, \ C < x < D, \) or \( F < x. \)
   For the energy \( E_2 \), motion is restricted to
   \( G < x. \)
   (b) The particle will remain at rest when \( F = -\frac{dU}{dx} \), which corresponds to \( H, I, J, \) or \( K. \)
   (c) Motion within turning points is possible for \( E < E_y. \)
   The positions will be in the valleys.
   (d) The equilibrium positions are where the slope is zero.
      Those at the bottom of the valleys, \( H \) and \( I, \) are stable; those at the peaks, \( I \) and \( K, \) are unstable.

33. We choose \( y = 0 \) at the release point. If we neglect air resistance, energy is conserved:
\[ E = \frac{1}{2}mv_t^2 + mgy_t = \frac{1}{2}mv_o^2 + mgy_o. \]
   (a) The kinetic energy depends on the speed, not the direction:
\[ \frac{1}{2}mv_y^2 + 0 = \frac{1}{2}mv_o^2 + mgh, \text{ or } v_y^2 = v_o^2 - 2gh, \text{ which gives } v_y = (v_o^2 - 2gh)^{1/2}. \]
   (b) For vertical motion, the speed at the highest point (where \( h = H \)) is zero:
\[ v_y = 0 = (v_o^2 - 2gh)^{1/2}, \text{ which gives } v_o = 2g(H)^{1/2}. \]
   (c) When the ball is thrown at an angle \( \theta, \) the speed at the highest point is \( v_y = v_{y_0} = v_0 \cos \theta. \) Here
\[ v_y^2 = v_o^2 - 2gh, \text{ or } v_{y_0}^2 = v_0^2 - 2gH. \]
Because \( v_0^2 = v_{y_0}^2 + v_{y_0}^2, \) we have
\[ v_{y_0}^2 = (v_0 \sin \theta)^2 = 2gH. \]
Thus \( v_o = (2gH)^{1/2}/\sin 45^\circ = 2g(H)^{1/2}. \)
40. With $y = 0$ at the bottom of the circle, we call the start point $A$, the bottom of the circle $B$, and the top of the circle $C$. From energy conservation we have

$$K_A + U_A = K_B + U_B = K_C + U_C.$$ (a) For the motion from $A$ to $B$:

$$mgH + 0 = 0 + \frac{1}{2}mv_B^2,$$ which gives $v_B = \sqrt{2gH}.$

(b) For the motion from $A$ to $C$:

$$mgH + 0 = mg(2R) + \frac{1}{2}mv_C^2,$$ which gives $v_C = \sqrt{2g(2R)}.$

(c) At the top of the circle we have the forces $mg$ and $N$, both downward, that provide the centripetal acceleration:

$$mg + N = mv_C^2/R,$$ which gives

$$N = m(v_C^2/R - g) = m[2g(2R)/R - g] = \frac{mg}{R} (2H/R - 5).$$

(d) The minimum value of $N$ is zero; since the track can only push on the car. Thus

$$2H_{min}/R - 5 = 0,$$ which gives $H_{min} = \frac{5R}{2}.$

The speed at $C$ will be

$$v_{C_{min}} = \sqrt{2g[H_{min} - 2R]} = \sqrt{5R}.$$

41. (a) We take the gravitational potential energy to be 0. The spring potential energy depends on the

amount of compression or extension of the spring.

At the position $x$, the length of the spring is $\sqrt{h^2 + x^2}$. The spring potential energy is

$$U(x) = \frac{1}{2}k\left(\sqrt{h^2 + x^2} - L\right)^2.$$

(b) We find the force produced by the spring by differentiating:

$$F = -\frac{dU}{dx} = -\frac{1}{2}k(x)(h^2 + x^2 - L^2) \frac{2x}{h^2 + x^2} = -k\frac{x}{h^2 + x^2}(h^2 + x^2 - L).$$

42. Because $U(r) = U_0[(r_0/r)^{12} - 2(r_0/r)^6]$ depends only on the separation, we find the force from

$$F_r = -\frac{dU}{dr} = -(12U_0/r_0^{12})(-12r_0^{11} - 12r_0^5(-6/r^7)) = (12U_0/r_0)[(r_0/r)^{13} - (r_0/r)^7].$$

The force will be zero when $(r_0/r)^{13} = (r_0/r)^7$, which gives $(r_0/r)^6 = 1$, or $r = \frac{r_0}{2}$.

At this separation, the potential energy is $U = U_0(1 - 2) = -U_0$.

43. (a) We find the potential energy for the force $F(x) = -ax + bx^2$ from

$$U(x) = U(0) - \int_0^x \left[-ax + bx^2\right] \, dx = U(0) + ax^2 - \frac{bx^3}{3} = 3 + \frac{3x^2}{2} - \frac{0.2x^3}{3},$$ with $x$ in m.
55. (a) Because \( F(x) = ax + bx^3 + cx^4 \) is a one-dimensional force that depends only on position, it is conservative.

(b) To test \( F = Ax^2 \hat{i} + Bxy \hat{j} \), we find the work for a displacement from \((0, 0)\) to \((1, 1)\) for the two paths indicated in the diagram:

\[
W_1 = \int_0^1 F_x \, dx + \int_0^1 F_y \, dy = \int_0^1 Ax^2 \, dx + \int_0^1 Bxy \, dy
\]

\[
= \frac{1}{2} A x^3 \bigg|_0^1 + \frac{1}{4} B x^2 y \bigg|_0^1 = \left( \frac{A}{3} - 0 \right) + \left( \frac{B}{2} - 0 \right) = \frac{A}{3} + \frac{B}{2}
\]

\[
W_II = \int_0^1 F_y \, dy + \int_0^1 F_x \, dx = \int_0^1 Bxy \, dy + \int_0^1 Ax^2 \, dx
\]

\[
= \frac{1}{2} B x^2 y \bigg|_0^1 + \frac{3}{4} A x^3 \bigg|_0^1 = \left( 0 - 0 \right) + \left( \frac{A}{3} - 0 \right) = \frac{A}{3}
\]

Because the work depends on the path, the force is not conservative.

60. We choose \( y = 0 \) at the bottom of the loop.

With no friction, energy is conserved.

The initial (and constant) energy is

\[
E_i = m g y_i = \frac{1}{2} m v_i^2
\]

\[
= (0.050 \text{ kg})(9.8 \text{ m/s}^2)(10 \times 10^{-2} \text{ m}) + 0 = 4.9 \times 10^{-2} \text{ J}
\]

(a) \( E_i = 0 + \frac{1}{2} m v_i^2 \)

\[
= \frac{1}{2}(0.050 \text{ kg}) v_i^2 = 4.9 \times 10^{-2} \text{ J}, \quad \text{which gives} \quad v_i = 1.4 \text{ m/s}
\]

\[
E_r = m g y_r = \frac{1}{2} m v_r^2
\]

\[
= (0.050 \text{ kg})(9.8 \text{ m/s}^2)(8 \times 10^{-2} \text{ m}) + \frac{1}{2}(0.050 \text{ kg}) v_i^2
\]

\[
= 4.9 \times 10^{-2} \text{ J}, \quad \text{which gives} \quad v_r = 1.63 \text{ m/s}
\]

\[
E_f = m g y_f = \frac{1}{2} m v_f^2
\]

\[
= (0.050 \text{ kg})(9.8 \text{ m/s}^2)(12 \times 10^{-2} \text{ m}) + \frac{1}{2}(0.050 \text{ kg}) v_i^2 = 4.9 \times 10^{-2} \text{ J}, \quad \text{which gives} \quad v_f^2 < 0.
\]

Thus \( v_r \) is not possible and the particle never reaches point \( d \).

(b) At the highest point the particle has no kinetic energy, so it must have the same potential energy as the initial point: \( y_{\text{max}} = y_f = 0.05 \text{ m} \).

64. We choose \( y = 0 \) at the lowest position. At the top of the swing the tension and the weight must provide the centripetal acceleration;

\[
T + m g = m a_c / h
\]

The tension must pull on the mass, so we have

\[
T = m a_c / (h + g) = 0, \quad \text{or} \quad a_c = -g
\]

Because the tension does not work, we can apply conservation of energy from the release point to the position of the mass directly above the nail:

\[
K_f + U_f = K_i + U_i
\]

\[
0 + m g L = \frac{1}{2} m v_f^2 + m g (2 h), \quad \text{or} \quad v_f^2 = 2 g (L - 2 h)
\]

From the condition on the tension, we have

\[
2 \mu (L - 2 h) \geq g h, \quad \text{which gives} \quad h \leq 2 L / (5 \mu) \geq 0.4 \text{ m}
\]

68. Because the tension in the rope is perpendicular to the motion, it does no work.

(a) From the work-energy theorem we have

\[
W_{\text{net}} = W_f - W_i = \Delta K = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2
\]

\[
W_f = \frac{1}{2}(1.8 \text{ kg})(2.1 \text{ m/s})^2 - \frac{1}{2}(1.8 \text{ kg})(3.5 \text{ m/s})^2 = -7.1 \text{ J}
\]

(b) Friction opposes the tangential motion.

\[
W_f = -\mu m g s = -\mu m g (2 a r)
\]

\[-7.1 = -\mu (1.8 \text{ kg})(9.8 \text{ m/s}^2)2 \pi (0.31 \text{ m})], \quad \text{which gives} \quad \mu = 0.21
\]

(c) If we take the reference level at the table top, \( U = 0 \) at all times, since the table is horizontal.

(d) Because the work done by friction does not depend on the speed, \( W_f = -7.1 \text{ J} \) for each revolution.

If the block stops after \( n \) revolutions, we have

\[
W_{\text{total}} = \Delta K
\]

\[-7.1 n = 0 - \frac{1}{2}(1.8 \text{ kg})(3.5 \text{ m/s})^2, \quad \text{which gives} \quad n = 15 \text{ revolutions}
\]
70. (a) 

(b) Normally we would find \( r \) at which \( U = -Ae^{-kx} / r \) is minimum by setting \( dU/dr = 0 \):
\[
dU/dr = -Ae^{-kx} / r^2 + Ak e^{-kx} / r = Ae^{-kx} (1 + kr) / r^2 = 0,
\]
which gives \( r = \infty \), for which \( U \) is maximum (zero). The difficulty is that \( U \) is not defined for \( r < 0 \). From the plot we see that \( U \) is minimum \((-\infty)\) at \( r = 0 \).

(c) \( F(r) = -dU/dr = -Ae^{-kx} (1 + kr) / r^2 \), which is attractive.

(d) \[
\begin{align*}
[F(0.1)] &= Ae^{-0.1(1 + 0.1) / (0.1 \times 10^{-15} m^2 - 1.0 \times 10^3)] A \\
[F(10)] &= Ae^{-10(1 + 10) / (10 \times 10^{-15} m^2 - 5.0 \times 10^3)] A
\end{align*}
\]

75. We choose the coordinate system shown in the diagram, with \( y = 0 \) at the lowest point. Because the tension does no work from the release point to the point where the string is cut, we can use conservation of energy:

\[
K_i + U_i = K_f + U_f;
\]

\[
0 = mgL = \frac{1}{2}mv_0^2 + mgL(1 - \cos \alpha), \text{ which gives } v_0^2 = 2gL \cos \alpha.
\]

When the string is cut, the ball becomes a projectile with initial speed \( v_0 \) at an angle \( \alpha \) with the horizontal.

For the projectile motion, we have:
\[
x = x_0 + v_0 \cos \alpha \cdot t \quad \text{and} \quad y = y_0 + v_0 \sin \alpha \cdot t + \frac{1}{2}(-g)t^2.
\]

When the ball hits the floor, \( y = 0 \):
\[
0 = L(1 - \cos \alpha) + v_0 \sin \alpha \cdot t - \frac{1}{2}gt^2,
\]
from which we can find the time to hit the floor:
\[
t = \left[ \frac{(v_0 \sin \alpha)}{g} \right] \left[ -1 \pm \left( \frac{v_0 \sin \alpha}{g} \right)^2 + \frac{L}{g}(1 - \cos \alpha)^{1/2} \right].
\]

We take the positive value from the solution to the quadratic equation for \( t \).

The horizontal distance traveled in this time is:
\[
x = L \sin \alpha + v_0 \sin \alpha \cdot t \;
\]

\[
= L \sin \alpha + \left( v_0 \cos \alpha \right) \left( \frac{v_0 \sin \alpha}{g} \right) \sin \alpha + \left( v_0 \cos \alpha \right) \left( \frac{L}{g} \right) \sin \alpha \left( \frac{v_0 \sin \alpha}{g} \right) + (L/g)(1 - \cos \alpha)^{1/2}.
\]

When we use \( v_0^2 = 2gL \cos \alpha \) and \( \sin^2 \alpha = 1 - \cos^2 \alpha \) and do some algebra, we get:
\[
x = L \sin \alpha + 2 \sin \alpha \cos \alpha + 2(\cos^2 \alpha - \cos^2 \alpha)(1/2).
\]