Chap. 8  Semiconductors

1. “direct band gap semiconductor”? “indirect band gap semiconductor”? 这对他们的应用有何影响？

2. ”direct band gap semiconductor”和“indirect band gap semiconductor”的吸收光谱有何不同？

3. Impurity orbits. Indium antimonide has $E_g=0.23$ eV; dielectric constant $\varepsilon=18$; electron effective mass $m_e=0.015m$. Calculate (a) the donor ionization energy; (b) the radius of the ground state orbit. (c) At what minimum donor concentration will appreciable overlap effects between the orbits of adjacent impurity atoms occur? This overlap tends to produce an impurity band- a band of energy levels which permit conductivity presumably by a hopping mechanism in which electrons move from one impurity site to a neighboring ionized impurity site.

4. Ionization of donors. In a particular semiconductor there are $10^{13}$ donors/cm$^3$ with an ionization energy $E_d$ of 1meV and an effective mass 0.01$m$. (a) Estimate the concentration of conduction electrons at 4 K. (b) What is the value of the Hall coefficient? Assume no acceptor atoms are present and that $E_g>>k_BT$.

5. Hall effect with two carrier types. Assuming concentrations $n, p$; relaxation times $\tau_e, \tau_h$; and masses $m_e, m_h$, show that the Hall coefficient in the drift velocity approximation

$$ R_H = \frac{1}{e} \frac{p-nb^2}{(p+nb)^2}, $$

where $b=m_e/m_h$ is the mobility ratio. In the derivation neglect terms of order $B^2$.

Hint: In the presence of a longitudinal electric field, find the transverse electric field such that the transverse current vanishes.

Hint: use

$$ \begin{pmatrix} j_x \\ j_y \\ j_z \end{pmatrix} = \frac{\sigma_0}{1+(\omega_c\tau)^2} \begin{pmatrix} 1 & -\omega_c\tau & 0 \\ \omega_c\tau & 1 & 0 \\ 0 & 0 & 1+(\omega_c\tau)^2 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}, $$

but for two carrier types; neglect $(\omega_c\tau)^2$ in comparison with $\omega_c\tau$.

6. Magnetoresistance with two carrier types. Consider a conductor with a concentration $n$ of electrons of effective mass $m_e$ and relaxation time $\tau_e$; and a concentration $p$ of holes of effective mass $m_h$ and relaxation time $\tau_h$. Treat the limit of very strong magnetic fields, $\omega_c\tau>>1$. (a) Show in the limit that $\sigma_{xy}=(n-p)e/B$. (b) Show that the Hall field is given by, with $Q=\omega_c\tau$,

$$ E_y = -(n-p)\left( \frac{n}{Q_e} + \frac{p}{Q_h} \right)^{-1} E_x, $$
which vanishes if \( n=p \). (e) Show that the effective conductivity in the \( x \) direction is
\[
\sigma_{\text{eff}} = \frac{e B}{\left( n \frac{Q_e}{Q_n} + \frac{p}{Q_p} \right) + (n-p)^2 \left( n \frac{Q_e}{Q_n} + \frac{p}{Q_p} \right)^{-1}}.
\]
If \( n=p \), \( \sigma \propto B^{-2} \). If \( n \neq p \), \( \sigma \) saturates in strong fields; that is it approaches a limit independent of \( B \) as \( B \to \infty \).

7. Spin-orbit effect 對一些半導體的價電葉頂端 \( k=0 \) 附近結構有何影響？

8. 何謂“heavy hole”? 何謂“light hole”? 何謂”split-off hole“？

9. Band edge structure on \( k\cdot p \) perturbation theory. Consider a nondegenerate orbital \( \psi_{nk} \) at \( k=0 \) in the band \( n \) of a cubic crystal. Use second-order perturbation theory to find the result
\[
\varepsilon_n(k) = \varepsilon_n(0) + \frac{\hbar k^2}{2m} + \frac{\hbar^2}{m^2} \sum_j \frac{\langle n| k \cdot p | j \rangle^2}{\varepsilon_n(0) - \varepsilon_j(0)},
\]
where the sum is over all other orbitals \( \psi_{jk} \) at \( k=0 \). The effective mass at this point is
\[
\frac{m}{m^*} = 1 + \frac{2}{m} \sum_j \frac{\langle n| k \cdot p | j \rangle^2}{\varepsilon_n(0) - \varepsilon_j(0)}.
\]
The mass at the conduction band edge in a narrow gap semiconductor is often dominated by the effect of the valence band edge, whence
\[
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\]
The mass at the conduction band edge in a narrow gap semiconductor is often dominated by the effect of the valence band edge, whence
\[
\frac{m}{m^*} \approx \frac{2}{mE_g} \sum_\nu \langle \psi | \nu \rangle^2,
\]
where the sum is over the valence bands; \( E_g \) is the energy gap. For given matrix elements, small gaps lead to small masses.

10. 解釋何謂“pseudopotential”?

11. At room temperature (300K) the effective density of states in the valence band is \( 1.04 \times 10^{19} \text{ cm}^{-3} \) for silicon and \( 7 \times 10^{18} \text{ cm}^{-3} \) for gallium arsenide, find the corresponding effective masses of holes. Compare these masses with the free-electron mass.

12. Calculate the location of \( E_i \) in silicon at liquid nitrogen temperature (77K), at
room temperature (300K), and at 100°C (let \( m_p = 0.5 m_0 \) and \( m_n = 0.3 m_0 \)). Is it reasonable to assume that \( E_f \) is in the center of the forbidden gap?

13. Calculate the Fermi level (chemical potential) of silicon doped with \( 10^{15} \), \( 10^{17} \), and \( 10^{19} \) phosphorus atoms/cm\(^3\) at room temperature, assuming complete ionization. From the calculated Fermi level, check if the assumption of complete ionization is justified for each doping.

14. Please plot the electron density as a function of \( 1000/T \) for a silicon sample with \( 10^{16} \) cm\(^{-3}\) n-type doping atoms. Please estimate the temperature for the electron density of the system becoming saturated from intrinsic, and from saturated to frozen-out regime. Assume the ionization energy of the donor is 5 meV.

15. Explain what is (a) zone folding (b) Bloch oscillation (c) type-II band alignment (d) modulation doping (e) reduced zone scheme (f) exchange energy (g) self-consistent band calculation (h) local density approximation (i) crystal momentum

16. Explain why the projection of the trace of the motion on the x-y plane for an electron moving in a uniform magnetic field B in the z direction has the same shape as the trace in the k-space but with 90-degree rotation? What is the ratio of the area between the closed shape in real space and that in k-space?

17. Assume the electron in a cubic structure has the \( \varepsilon \)-k relation:
\[
\varepsilon_k = -\alpha - 2\gamma_1 \cos k_x a - 2\gamma_2 \cos k_y a - 2\gamma_3 \cos k_z a.
\]

What is the energy bandwidth? Write down the inversed effective mass matrix
\[
\left( \frac{1}{m} \right)
\]
for electrons near the zone center.

18. Assume near a top of a valence band of a semiconductor, the \( \varepsilon \)-k relation can be described by:
\[
\varepsilon_k = -\alpha - 8\gamma_1 k_x a \cos \frac{1}{2} k_y a \cos \frac{1}{2} k_z a.
\]

If an electron with \((k_x, k_y, k_z) = (\frac{1}{20}, \frac{1}{20}, \frac{1}{20})\) \( a \) is missing, what is the crystal momentum, effective mass, and group velocity of the resulting hole.