4.55

55

\[ R_e = \frac{V_T}{I_e} \cdot \frac{25 \text{ mV}}{0.8 \text{ mA}} \cdot 31.25 \Omega \]

\[ \beta = \frac{r_e}{1 - \beta} = \frac{0.99}{0.01} = 99 \]

\[ r_T = (\beta + 1) \cdot R_e = 3.125 \text{ k}\Omega \]

4.61

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(a) Using the voltage divider rule,

\[ \frac{V_C}{V_B} = \frac{R_e}{R_e + r_T} \quad \text{Q.E.D.} \]

(b) Node equation at \( B \),

\[ i_b = \frac{V_{be}}{r_e} - g_m V_{be} \]

\[ \quad = \frac{V_{be}}{r_e} \left( 1 - g_m R_e \right) \]

\[ \quad = \frac{V_{be}}{r_e} \left( 1 - g_m \frac{0}{g_m} \right) \]

\[ \quad = \frac{V_{be}}{r_e} \left( 1 - \alpha \right) \]

\[ \quad = \frac{V_{be}}{r_e} \left( 1 - \frac{\beta}{\beta + 1} \right) \]

\[ \quad = \frac{V_{be}}{r_e} \frac{1}{\beta + 1} \]
But from voltage-divider rule
\[ V_{be} = V_b \frac{R_e}{R_e + R_e} \]

Thus
\[ I_b = \frac{1}{(\beta + 1)R_e} \frac{V_b R_e}{R_e + R_e} \]

from which we find
\[ R_{in} \equiv \frac{V_b}{I_b} = (\beta + 1)(R_e + R_e) \] Q.E.D.

For \( R_e = 1 \, k\Omega \), \( \beta = 100 \), and \( I_e = 1 \, mA \),
\[ V_e = \frac{V_T}{I_e} = \frac{25 \, \text{mV}}{1 \, \text{mA}} = 25 \, \Omega \]

Thus,
\[ \frac{V_e}{V_b} = \frac{1000}{1000 + 25} = 0.976 \, \text{V/V} \]
\[ R_{in} = (100 + 1)(1000 + 25) \, \Omega \]
\[ = 101 \times 1.025 \, k\Omega \]
\[ = 103.5 \, k\Omega \]

Refer to Fig. P4.64.

For large \( \beta \), the dc base current will be negligibly small. Thus the dc voltage at the base can be found directly using the voltage-divider rule,
\[ V_B = 15 \frac{100}{180 + 100} = 7.5 \, V \]
Assuming \( V_{BE} \approx 0.7 \, V \),
\[ V_E = 7.5 - 0.7 = 6.8 \, V \]

Thus,
\[ I_E = \frac{6.8 \, V}{6.8 \, k\Omega} = 1 \, mA \]
Thus, using the voltage-divider rule we find the

\[ \frac{V_{be}}{V_i} = \frac{R}{R_e + R} \quad \text{Q.E.D.} \]

Also,

\[ i_c = \frac{V_b}{R_e + R} = \frac{V_i}{R_e + R} \]

and,

\[ V_{a2} = -a \cdot i_c \cdot R_c \]

\[ = -a \cdot \frac{R_c V_i}{R_e + R} \]

Thus,

\[ \frac{V_{a2}}{V_i} = -a \cdot \frac{R_c}{R_e + R} \quad \text{Q.E.D.} \]

Substituting \( R_e = V_i / I_e = 25 \Omega \), \( R_e = 6.8 \ \Omega \), \( R_c = 4.3 \ \Omega \) and \( a = 1 \) gives

\[ \frac{V_{a2}}{V_i} = \frac{6.8}{0.025 + 6.8} = 0.996 \ \text{V/V} \]

\[ \frac{V_{a2}}{V_i} = -\frac{4.3}{6.8 + 0.025} = 0.63 \ \text{V/V} \]

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Equation of line \( L_1 \):

\[ i_C = 5 + \frac{5}{100} V_{CE}, \ \text{mA} \]

\[ \therefore i_C = 5 + 0.05 V_{CE} \]
Equation of load line:
\[ I_C = \frac{V_{CC} - V_{CE}}{R_C} \]

i.e. \[ I_C = 10 - V_{CE} \] (2)

Solving (1) together with (2) yields for the bias point Q:
\[ I_C = \frac{5.24 \text{ mA}}{V_{CE} = 4.76 \text{ V}} \]

Now for a signal of 30-\( \mu \text{A} \) peak superimposed on \( I_B = 50 \mu \text{A} \), the operating point moves along the load line between points N and M. To obtain the coordinates of point M, we solve Eq. (2) simultaneously with the equation of line \( L_2 \), which is
\[ I_C = 8 + \frac{8}{100} V_{CE} \]
to obtain
\[ I_C |_M = 8.15 \text{ V} \]
\[ V_{CE} |_M = 1.85 \text{ V} \]

Similarly, the coordinates of point N can be found by solving Eq. (2) simultaneously with the equation for line \( L_3 \),
\[ I_C = 2 + \frac{2}{100} V_{CE} \]
to obtain
\[ I_C |_N = 2.16 \text{ mA} \]
\[ V_{CE} |_N = 7.84 \text{ V} \]

Thus the resulting collector current signal has a positive peak of \( 8.15 - 5.24 = 2.91 \text{ mA} \) and a negative peak of \( 5.24 - 2.16 = 3.08 \text{ mA} \) for 5.99 mA peak-to-peak. The corresponding collector voltage signal has a positive peak of \( 7.84 - 4.76 = 3.08 \text{ V} \) and a negative peak of \( 4.76 - 1.85 = 2.91 \text{ V} \), for a peak-to-peak signal of 5.99 V.
Assuming a very large $\beta$, $\alpha = 1$, and the base current can be neglected,

$$I_E R_E = \frac{1}{3} V_{CC} = \frac{1}{3} \times 9 = 3 \text{ V}$$

For $I_E = 0.5 \text{ mA}$,

$$R_E = \frac{6}{3} \Omega$$

$$I_C R_C = \frac{1}{3} V_{CC} = 3 \text{ V}$$

For $I_C \approx I_E = 0.5 \text{ mA}$,

$$R_C = \frac{6}{3} \Omega$$

Now for the voltage divider $R_1, R_2$, if we neglect the base current,

$$\frac{V_{CC}}{R_1 + R_2} = 0.2 \times 0.3 = 0.1 \text{ mA}$$

$$\Rightarrow R_1 + R_2 = 90 \text{ k}\Omega,$$

and to obtain a voltage at the base of $V_E + V_{BE} = 3 + 0.7 = 3.7 \text{ V},$

$$\frac{R_E}{R_1 + R_2} = \frac{3.7}{9}$$

$$\Rightarrow R_2 = \frac{3.7}{9} \times 90 = 37 \text{ k}\Omega$$

Thus, $R_1 = 90 - 37 = 53 \text{ k}\Omega$

Now for a BJT with $\beta = 100$,

$$I_E = \frac{V_{BB} - V_{BE}}{R_E + \frac{R_B}{\beta + 1}}$$

where $V_{BB} = 9 \frac{R_2}{R_1 + R_2} = 3.7 \text{ V},$

and $R_B = R_1 // R_2 = \frac{53 \times 37}{218} = 21.8 \text{ k}\Omega$

Thus, $I_E = \frac{3.7 - 0.7}{6 + \frac{21.8}{101}} = 0.48 \text{ mA}$
\[ I_E = \frac{V_{BB} - V_{BE}}{R_E + R_B/(\beta + 1)} \]

where
\[ V_{BB} = V_{CC} \left( \frac{R_2}{R_1 + R_2} \right) = 9 \left( \frac{15}{27 + 15} \right) = 3.21 \text{ V} \]
\[ R_B = R_1 \parallel R_2 = 15 \parallel 27 = 9.64 \text{ k}\Omega \]

Then,
\[ I_E = \frac{3.21 - 0.7}{1.2 + \frac{9.64}{101}} = 1.94 \text{ mA} \]

\[ G_m = \frac{I_C}{V_T} = \frac{0.49 \times 1.94}{0.025} = 76.8 \text{ mA/V} \]

\[ \beta = \frac{V_T}{V_T} = \frac{100}{76.8} = 1.3 \text{ k}\Omega \]
\[ \tau = \frac{V_T}{I_C} = \frac{100}{0.49 \times 1.94} = 52.1 \text{ k}\Omega \]

\[ R_i = R_B \parallel R_T = 9.64 \parallel 1.3 = 1.15 \text{ k}\Omega \]
\[ G_m = -G_m = -76.8 \text{ mA/V} \]
\[ R_o = R_C \parallel \tau_0 \]
\[ = 2.2 \parallel 52.1 = 2.11 \text{ k}\Omega \]

\[ A_V \equiv \frac{V_o}{V_i} = \frac{V_o}{V_i} \]
\[ = \frac{R_i}{R_i + R_T} \cdot \frac{G_m}{(R_o \parallel R_T)} \]
\[ = -\frac{1.15}{10 + 1.15} \times 76.8 \times (2.11 / 2) \]
\[ = -8.13 \text{ V/V} \]

\[ A_i \equiv \frac{i_o}{i_i} = \frac{V_o / R_L}{V_T / (R_o + R_i)} \]
\[ = \frac{V_o}{U_T} \cdot \frac{R_1 + R_2}{R_L} \]
\[ = -8.13 \times \frac{10 + 1.15}{2} \]
\[ = -45.3 \text{ A/A} \]
Refer to Fig. P4.78.

\[ V_{BB} = 9 \frac{47}{82+47} = 3.28 \text{ V} \]

\[ R_B = \frac{47}{182} = 21.88 \text{ k}\Omega \]

\[ I_E = \frac{3.28 - 0.7}{3.6 + \frac{25.88}{101}} = 0.66 \text{ mA} \]

\[ I_C = 0.99 \times 0.66 = 0.63 \text{ mA} \]

\[ g_m = \frac{0.65}{0.025} = 26 \text{ mA/V} \]

\[ r_t = \frac{100}{26} = 3.85 \text{ k}\Omega \]

\[ r_e = \frac{100}{0.66} = 151.5 \text{ k}\Omega \]

\[ A_v = \frac{V_o}{V_i} = \frac{3.41}{10 + 3.41} = 0.31 \]

\[ = -10.1 \text{ V/V} \]

which is about 25% higher than in the original design. The improvement is not as large as might have been expected because although \( R_1 \) increases, \( I_m \) decreases by about the same factor. Indeed most of the improvement is due to the increase in \( R_e \) and hence in the effective load resistance.
Refer to Fig P.14.82

(a) \[ I_E = \frac{15 - 0.7}{R_E + \frac{R_3}{\beta + 1}} \]

\[ 1 = \frac{14.3}{R_E + \frac{2.5}{101}} \]

\[ \Rightarrow R_E \approx 14.3 \, \Omega \]

(b) \[ V_C = 15 - R_C I_C \]
\[ 5 = 15 - R_C \times 0.19 \times 1 \]
\[ \Rightarrow R_C = 10 \, \Omega \]

(c) \[ g_m = \frac{I_C}{V_T} \approx \frac{1}{0.025} = 40 \, \text{mA/V} \]

\[ r_m = \frac{B}{g_m} = \frac{100}{40} = 2.5 \, \Omega \]

\[ r_o = 100 \, \Omega \]

\[ A_v = \frac{V_o}{V_S} = \frac{V_T}{V_T} = \frac{V_o}{V_T} \]

\[ = \frac{r_m}{R_3 + r_m} \times -g_m (\frac{r_o}{R_C \parallel R_L}) \]
\[ = -\frac{2.5}{2.5 + 2.5} \times 40 (100 \parallel 10 \parallel 5) = -64.5 \, \text{V/V} \]

8-8